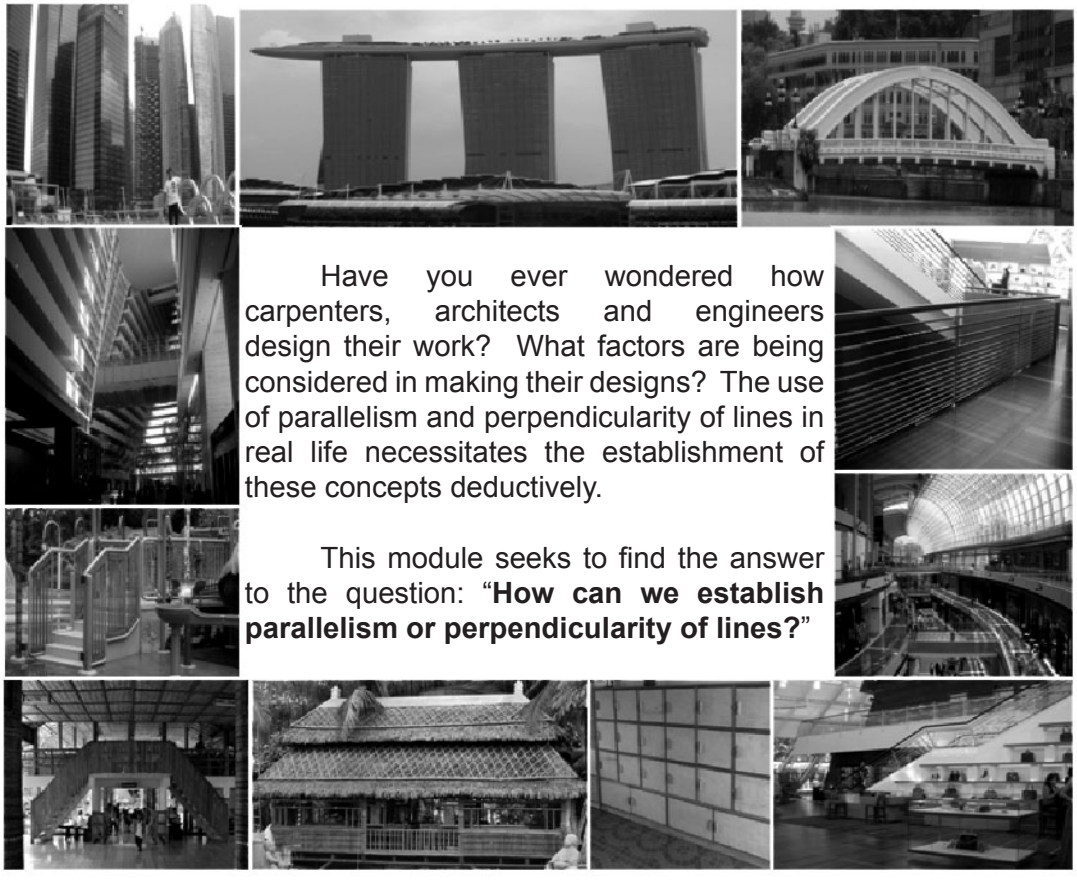
9 PARALLELISM AND PERPENDICULARITY

I. INTRODUCTION AND FOCUS QUESTIONS



Have you ever wondered how carpenters, architects and engineers design their work? What factors are being considered in making their designs? The use of parallelism and perpendicularity of lines in real life necessitates the establishment of these concepts deductively.

This module seeks to find the answer to the question: "**How can we establish parallelism or perpendicularity of lines?**"

II. LESSONS AND COVERAGE

In this module, you will examine this question when you study the following:

Lesson 1 – Parallelism and Perpendicularity

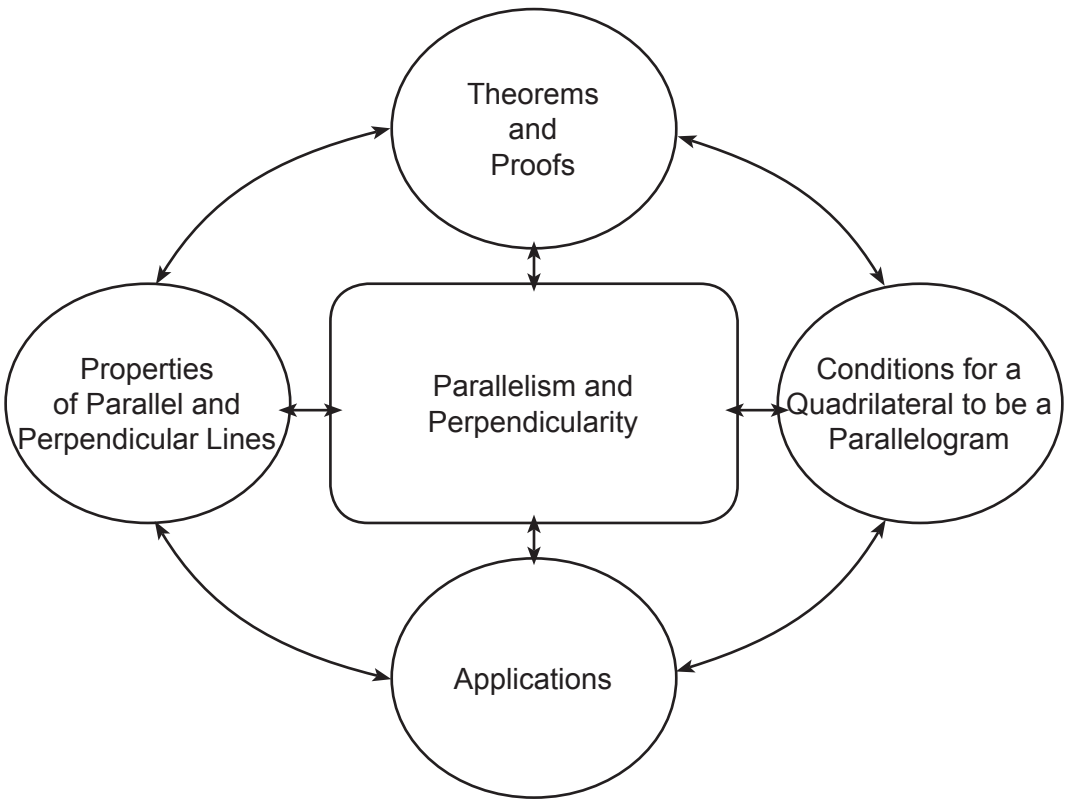
1.1 Proving Theorems on Parallel and Perpendicular Lines
1.2 Proving Properties of Parallel Lines Cut by a Transversal
1.3 Conditions to Prove that a Quadrilateral is a Parallelogram
1.4 Applications of Parallelism and Perpendicularity

In this lesson, you will learn to:

- illustrate parallel and perpendicular lines;
- demonstrate knowledge and skills involving angles formed by parallel lines and transversals;
- determine and prove the conditions under which lines and segments are parallel or perpendicular;
- determine the conditions that make a quadrilateral a parallelogram and prove that a quadrilateral is a parallelogram and;
- use properties to find measures of angles, sides, and other quantities involving parallelograms.

Module Map

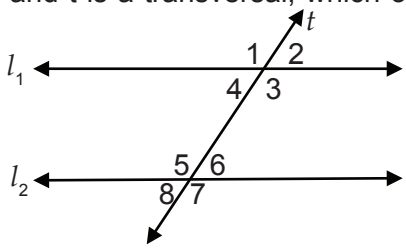
Here is a simple map of the lesson that will be covered in this module.



III. PRE - ASSESSMENT

Find out how much you already know about this module. Choose the letter that corresponds to the best answer and write it in a separate sheet. Please answer all items. After taking this short test, take note of the items that you were not able to answer correctly. Correct answers are provided as you go through the module.

(K)1. Using the figure below, if l_1 || l_2 and t is a transversal, which of the following are corresponding angles?

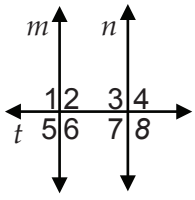


a. ∠4 and ∠6, ∠3 and ∠5
b. ∠1 and ∠7, ∠2 and ∠8
c. ∠1 and ∠5, ∠2 and ∠6
d. ∠4 and ∠5, ∠3 and ∠6

(K)2. All of the following are properties of a parallelogram except:

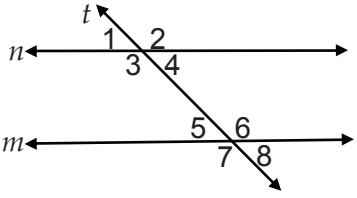
a. Diagonals bisect each other.
b. Opposite angles are congruent.
c. Opposite sides are congruent.
d. Opposite sides are not parallel.

(K)3. Lines m and n are parallel cut by transversal t which is also perpendicular to m and n. Which statement is not correct?



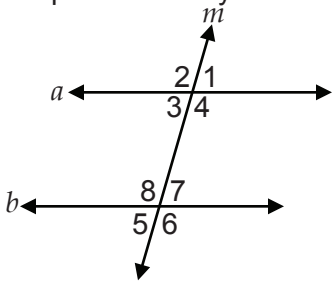
a. ∠1 and ∠6 are congruent.
b. ∠2 and ∠3 are supplementary.
c. ∠3 and ∠5 are congruent angles.
d. ∠1 and ∠4 form a linear pair.

(K)4. Using the figure below, which of the following guarantees that m || n?



a. ∠1 ≅ ∠7
b. ∠3 ≅ ∠5
c. ∠4 ≅ ∠5
d. ∠4 ≅ ∠7

(S)5. Lines a and b are parallel cut by transversal m. If $m\angle 1 = 85$, what is the measure of ∠5?

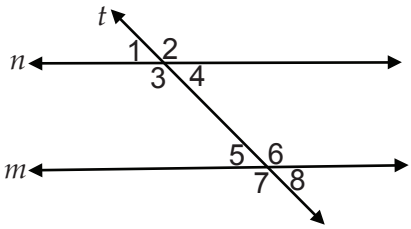


a. 80
b. 85
c. 95
d. 100

(S)6. ▭ JOSH is a parallelogram, $m\angle J = 57$, find the measure of $\angle H$.

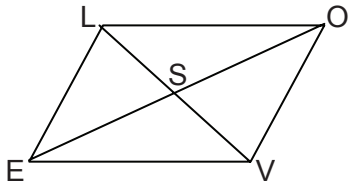
a. 43
b. 57
c. 63
d. 123

(S)7. Using the figure below, if $m \parallel n$ and t is a transversal which angles are congruent to $\angle 5$?



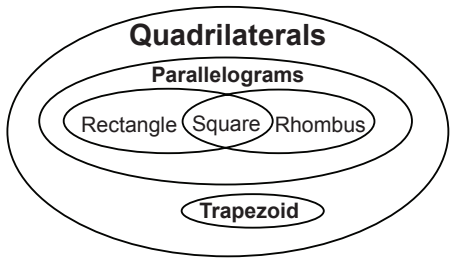
a. $\angle 1$, $\angle 2$ and $\angle 3$
b. $\angle 1$, $\angle 4$ and $\angle 8$
c. $\angle 1$, $\angle 4$ and $\angle 7$
d. $\angle 1$, $\angle 2$ and $\angle 8$

(S)8. ▭ LOVE is a parallelogram. If $SE = 6$, then what is SO?



a. 3
b. 6
c. 12
d. 15

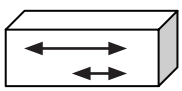
(U)9. The Venn Diagram below shows the relationships of quadrilaterals. Which statements are true?

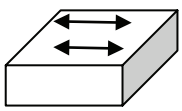


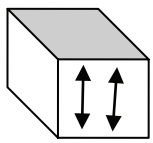
I - Squares are rectangles.
II- A trapezoid is a parallelogram.
III- A rhombus is a square.
IV- Some parallelograms are squares.

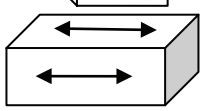
a. I and II
b. III and IV
c. I and IV
d. II and III

(U)10. All of the figures below illustrate parallel lines except:

a. 

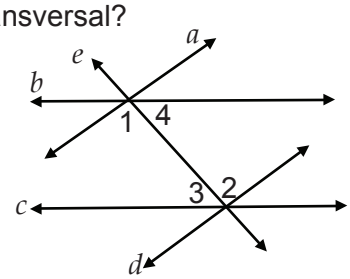
b. 

c. 

d. 

(U)11. In the figure below, $a \parallel d$ with e as the transversal. What must be true about $\angle 3$ and $\angle 4$, if $b \parallel c$ with e, also as the transversal?

a. $\angle 3$ is a complement of $\angle 4$.
b. $\angle 3$ is congruent to $\angle 4$.
c. $\angle 3$ is a supplement of $\angle 4$.
d. $\angle 3$ is greater than $\angle 4$.



(U)12. Which of the following statements ensures that a quadrilateral is a parallelogram?

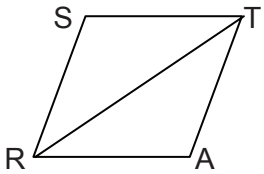
a. Diagonals bisect each other.
b. The two diagonals are congruent.
c. The consecutive sides are congruent.
d. Two consecutive angles are congruent.

(U) 13. Which of the following statements is always true?

a. Lines that do not intersect are parallel lines.
b. Two coplanar lines that do not intersect are parallel lines.
c. Lines that form a right angle are parallel lines.
d. Skew lines are parallel lines.

(U)14. $\square$ STAR is a rhombus with diagonal $\overline{RT}$, if $m\angle STR = 3x - 5$ and $m\angle ART = x + 21$. What is $m\angle RAT$?

a. 13
b. 34
c. 68
d. 112



(P)15. You are tasked to divide a blank card into three equal rows/pieces but you do not have a ruler. Instead, you will use a piece of equally lined paper and a straight edge. What is the sequence of the steps you are going to undertake in order to apply the theorem on parallel lines?

I – Mark the points where the second and third lines intersect the card.
II – Place a corner of the top edge of the card on the first line of the paper.
III – Repeat for the other side of the card and connect the marks.
IV – Place the corner of the bottom edge on the fourth line.

a. I, II, III, IV
b. II, III, IV, I
c. I, III, IV, II
d. II, IV, I, III

(P)16. You are a student council president. You want to request for financial assistance for the installation of a book shelf for the improvement of your school's library. Your student council moderator asked you to submit a proposal for their approval. Which of the following will you prepare to ensure that your request will be granted?

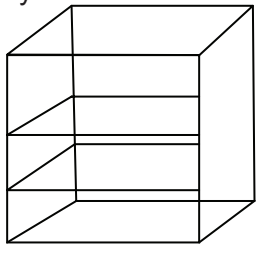
I. design proposal of the book shelf
II. research on the importance of book shelf
III. estimated cost of the project
IV. pictures of the different libraries

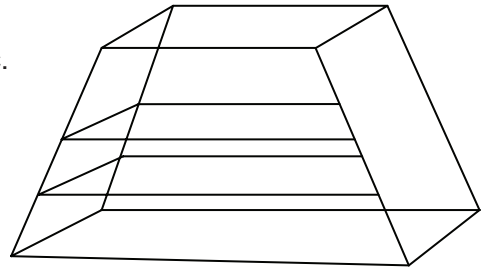
a. I only
b. I and II only
c. I and III only
d. II and IV only

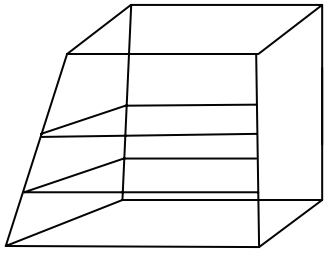
(P)17. Based on your answer in item no. 16, which of the following standards should be the basis of your moderator in approving or granting your request?

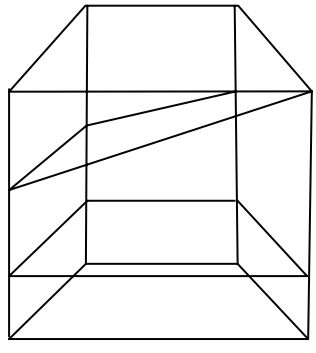
a. accuracy, creativity, and mathematical reasoning
b. practicality, creativity, and cost
c. accuracy, originality, and mathematical reasoning
d. organization, mathematical reasoning, and cost

(P)18. Based on item no. 16, design is common to all the four given options. If you were to make the design, which of the illustrations below will you make to ensure stability?

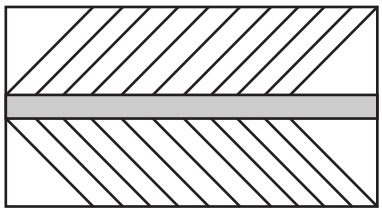
a. 

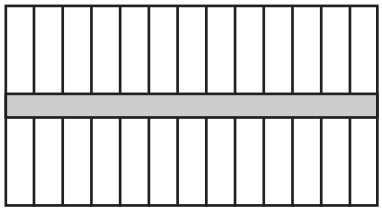
c. 

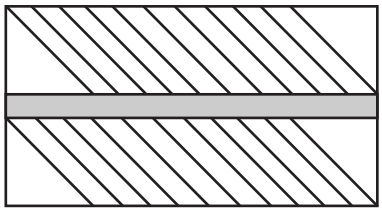
b. 

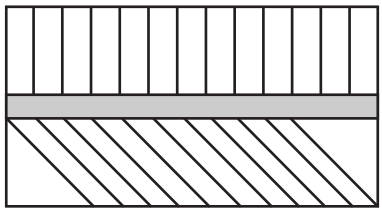
d. 

(P)19. You are an architect of the design department of a mall. Considering the increasing number of mall-goers, the management decided to restructure their parking lot so as to maximize the use of the space. As the head architect, you are tasked to make a design of the parking area and this design is to be presented to the mall administrators for approval. Which of the following are you going to make so as to maximize the use of the available lot?

a. 

b. 

c. 

d. 

(P)20. Based on your answer in item no. 19, how will your immediate supervisor know that you have a good design?

a. The design should be realistic.
b. The design should be creative and accurate.
c. The design should be accurate and practical.
d. The design shows a depth application of mathematical reasoning and it is practical.

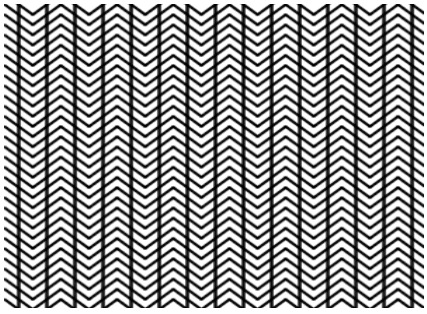
LEARNING GOALS AND TARGET:

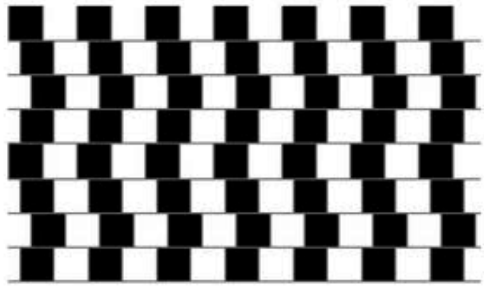
- The learner demonstrates understanding of the key concepts of parallel and perpendicular lines.
- The learner is able to communicate mathematical thinking with coherence and clarity in solving real-life problems involving parallelism and perpendicularity using appropriate and accurate representations.

What to Know

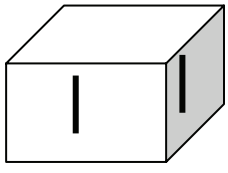
Start the module by taking a look at the figures below and then answer the succeeding questions.

Activity 1 OPTICAL ILLUSION





- Can you see straight lines in the pictures above? _____
- Do these lines meet/intersect? _____
- Are these lines parallel? Why? _____
- Are the segments on the faces of the prism below parallel? Why? _____
- Can you describe what parallel lines are? _____



- What can you say about the edges of the prism? _____
- Are these lines perpendicular? Why? _____
- Can you describe what perpendicular lines are? _____

You have just tried describing parallel and perpendicular lines. In Activities 2 and 3, your prior knowledge on parallelism and perpendicularity will be extracted.

Activity 2 GENERALIZATION TABLE

Direction: Fill in the first column of the generalization table below by stating your initial thoughts on the question.

"How can parallelism or perpendicularity of lines be established?"

My Initial Thoughts				

Activity 3 AGREE OR DISAGREE!

ANTICIPATION-REACTION GUIDE

Read each statement under the column TOPIC and write **A** if you agree with the statement; otherwise, write **D**.

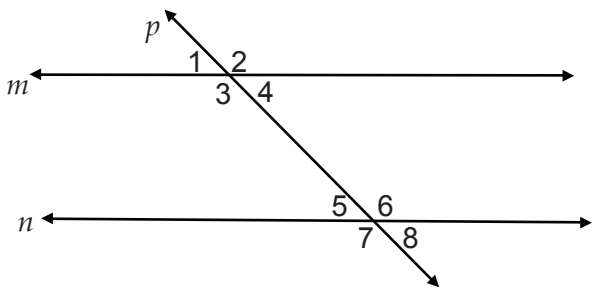
Before-Lesson Response	TOPIC: Parallelism and Perpendicularity
	1. Lines that do not intersect are parallel lines.
	2. Skew lines are coplanar.
	3. Transversal is a line that intersects two or more lines.
	4. Perpendicular lines are intersecting lines.
	5. If two lines are parallel to a third line, then the two lines are parallel.

	6. If two lines are perpendicular to the same line, then the two lines are parallel.
	7. If one side of a quadrilateral is congruent to its opposite side, then the quadrilateral is a parallelogram.
	8. Diagonals of a parallelogram bisect each other.
	9. Diagonals of a parallelogram are congruent.
	10. Diagonals of a parallelogram are perpendicular.
	11. Opposite sides of a parallelogram are parallel.
	12. Opposite angles of a parallelogram are congruent.
	13. Consecutive angles of a parallelogram are congruent.
	14. Squares are rectangles.
	15. Squares are rhombi.

Well, those were your thoughts and ideas about our lesson. Start a new activity to further explore on the important key concepts about parallel and perpendicular lines. I guess you have it already in your previous Math, but just to recall, I want you to answer the next activity.

Activity 4 NAME IT! A RECALL...

We see parallel lines everywhere. Lines on a pad paper, railways, edges of a door or window, fence, etc. suggest parallel lines. Complete the table below using the given figure as your reference:



Corresponding Angles	Alternate Interior Angles	Alternate Exterior Angles	Same Side Interior Angles	Same Side Exterior Angles

You gave your initial ideas on naming angle pairs formed by two lines cut by a transversal. What you will learn in the next sections will enable you to do the final project which involves integrating the key concepts of parallelism and perpendicularity of lines in model making of a book case. Now find out how these pairs of angles are related in terms of their measures by doing the first activity on investigating the relationship between the angles formed by parallel lines cut by a transversal.

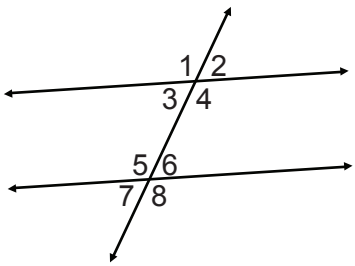
What to Process

Your goal in this section is to learn and understand key concepts of measurement of angles formed by parallel lines cut by a transversal and basic concepts about perpendicularity and the properties of parallelogram. Towards the end of this section, you will be encouraged to learn the different ways of proving deductively. You may also visit the link for this investigation activity. http://www.mathwarehouse.com/geometry/angle/interactive-transveral-angles.php

Activity 5 LET'S INVESTIGATE!

Two parallel lines when cut by a transversal form eight angles. This activity will lead you to investigate the relationship between and among angles formed.

Measure the eight angles using your protractor and list all inferences or observations in the activity.



$m\angle 1$= _____
$m\angle 2$= _____
$m\angle 3$= _____
$m\angle 4$= _____
$m\angle 5$= _____
$m\angle 6$= _____
$m\angle 7$= _____
$m\angle 8$= _____

OBSERVATIONS:

_____
_____
_____

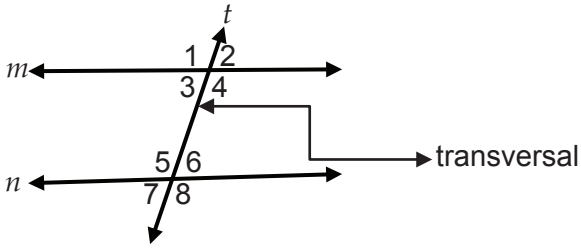
Now, think about the answers to the following questions. Write your answers in your answer sheet.

QUESTIONS ?

1. What pairs of angles are formed when two lines are cut by a transversal line?
2. What pairs of angles have equal measures? What pairs of angles are supplementary?
3. Can the measures of any pair of angles (supplementary or equal) guarantee the parallelism of lines? Support your answer.
4. How can the key concepts of parallel lines facilitate solving real-life problems using deductive reasoning?

Discussion: ***Parallelism***

1. Two lines are parallel if and only if they are coplanar and they do not intersect. ($m \parallel n$)



2. A line that intersects two or more lines at different points is called a **transversal**.

   a. The angles formed by the transversal with the two other lines are called:
      - *exterior angles* ($\angle 1$, $\angle 2$, $\angle 7$ and $\angle 8$)
      - *interior angles* ($\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$).

   b. The pairs of angles formed by the transversal with the other two lines are called:
      - *corresponding angles* ($\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$)
      - *alternate-interior angles* ($\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$)
      - *alternate-exterior angles* ($\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$)
      - *interior angles* on the same side of the transversal ($\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$)
      - *exterior angles* on the same side of the transversal ($\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$)

3. If two lines are cut by a transversal, then the two lines are parallel if:

   a. corresponding angles are congruent.
   b. alternate-interior angles are congruent.
   c. alternate-exterior angles are congruent.
   d. interior angles on the same side of the transversal are supplementary.
   e. exterior angles on the same side of the transversal are supplementary.

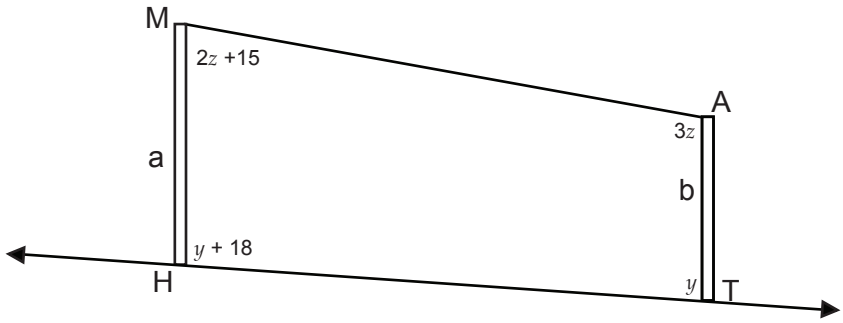
To strengthen your knowledge regarding the different angles formed by parallel lines cut by a transversal line and how they are related with one another, you may visit the following sites:

http://www.youtube.com/watch?v=AE3PqhlvqW0&feature=related
http://www.youtube.com/watch?v=VA92EWf9SRI&feature=relmfu

Activity 6 UNCOVERING THE MYSTERY OF PARALLEL LINES CUT BY A TRANSVERSAL

Study the problem situation below and answer the succeeding questions:

A zip line is a rope or a cable that you can ride down on a pulley. The pair of zip lines below goes from a 20- foot tall tower to a 15- foot tower 150 meters away in a slightly inclined ground as shown in the sketch. (Note: Tension of the rope is excluded.)



1. What kind of angle pairs are $\angle M$ and $\angle A$? $\angle MHT$ and $\angle ATH$?

   _____

   _____

2. Using the given information stated in the figure, what are the measures of the four angles?

Solution:	Answers:
	$m\angle M$ = _____ $m\angle A$ = _____ $m\angle MHT$ = _____ $m\angle ATH$ = _____

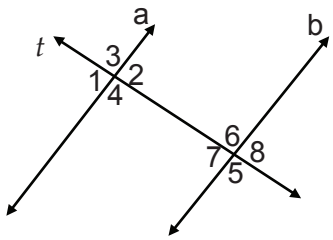
3. Are the two towers parallel? Why do you say so?

4. Is the zip line parallel to the ground? Why do you say so?

For practice you may proceed to this link:
http://www.regentsprep.org/Regents/math/geometry/GP8/PracParallel.htm

Activity 7 LINES AND ANGLES

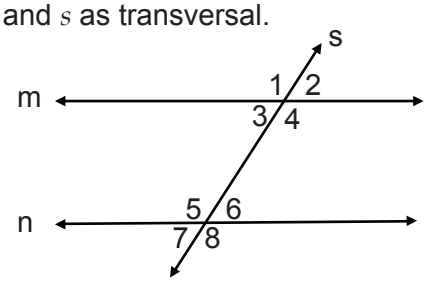
I. Study the figure and answer the following questions as accurate as you can. The figure below shows $a \parallel b$ with t as transversal.



Name:

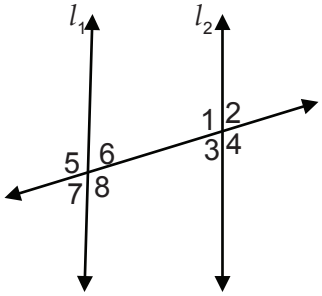
1. 2 pairs of corresponding angles _____ _____
2. 2 pairs of alternate interior angles _____ _____
3. 2 pairs of alternate exterior angles _____ _____
4. 2 pairs of interior angles on the same side of the transversal _____ _____
5. 2 pairs of exterior angles on the same side of the transversal _____ _____

II. Given $m \parallel n$ and s as transversal.



1. Name all the angles that are congruent to $\angle 1$. _____
2. Name all the angles that are supplement of $\angle 2$. _____

III. Find the value of x given that $l_1 \parallel l_2$.



1. $m\angle 1 = 2x + 25$ and $m\angle 8 = x + 75$ _____
2. $m\angle 2 = 3x - 10$ and $m\angle 6 = 2x + 45$ _____
3. $m\angle 3 = 4v - 31$ and $m\angle 8 = 2x + 7$ _____

Activity 8 AM I PERPENDICULAR? LET'S FIND OUT...!

Given any two distinct lines on a plane, the lines either intersect or are parallel. If two lines intersect, then they form four angles. Consider the figures below to answer the questions that follow.

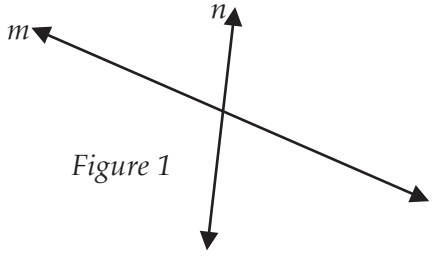


Figure 1

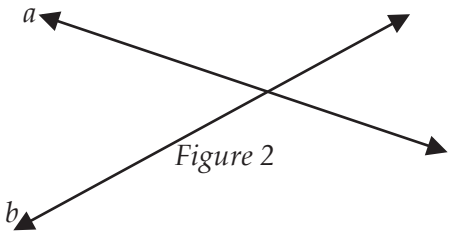


Figure 2

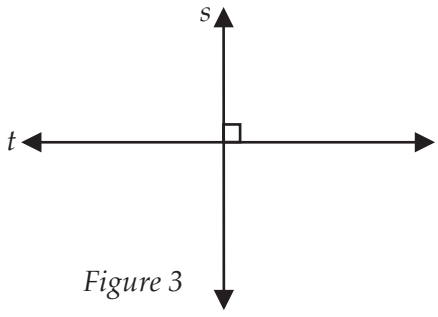


Figure 3

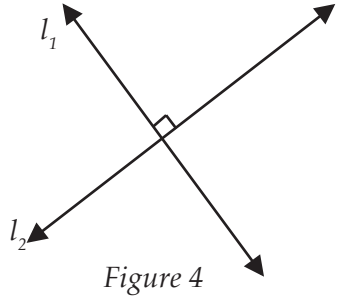


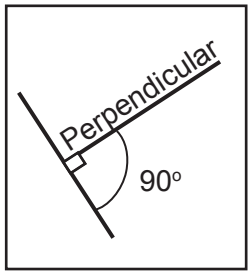
Figure 4

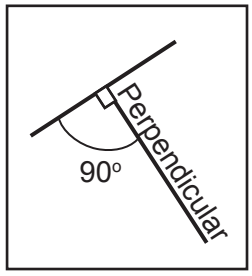
QUESTIONS ?

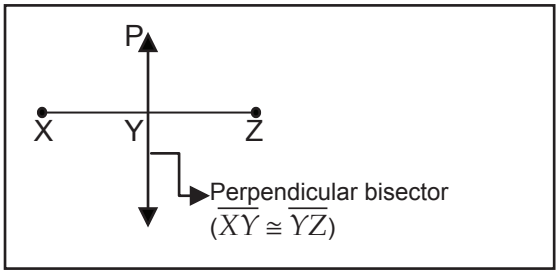
1. What is common in the four figures given above?
   _____
2. What makes figures 3 and 4 different from the first two figures?
   _____
3. What does this symbol ⌞ indicate?
   _____
4. Which among the four figures show perpendicularity? Check by using your protractor.
   _____
5. When are the lines said to be perpendicular to each other?
   _____
6. How useful is the knowledge on perpendicularity in real-life? Cite an example in which perpendicularity is said to be important in real-life.
   _____
   _____
   _____

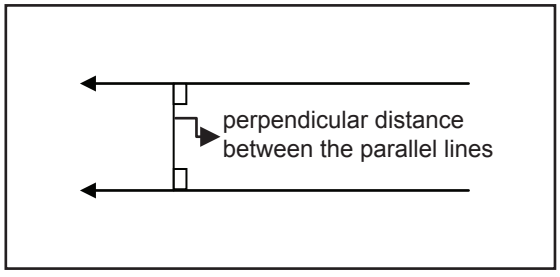
Discussion: ***Perpendicularity***

Two lines that intersect to form right angles are said to be **perpendicular**. This is not limited to lines only. Segments and rays can also be perpendicular. A **perpendicular bisector** of a segment is a line or a ray or another segment that is perpendicular to the segment and intersects the segment at its midpoint. The distance between two parallel lines is the **perpendicular distance** between one of the lines and any point on the other line.





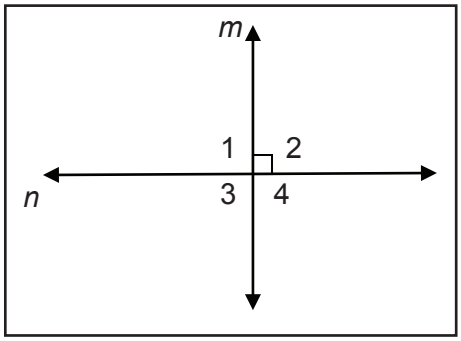


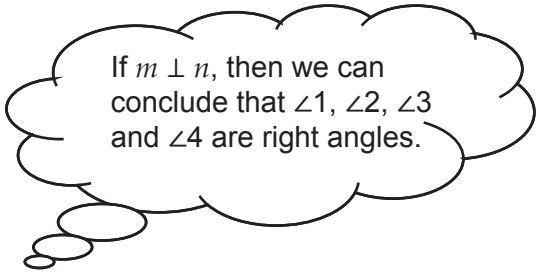


The small rectangle drawn in the corner indicates "right angle". Whereas, $\perp$ is a symbol use to indicate perpendicularity of lines as in $\overline{XZ} \perp \overline{PY}$.

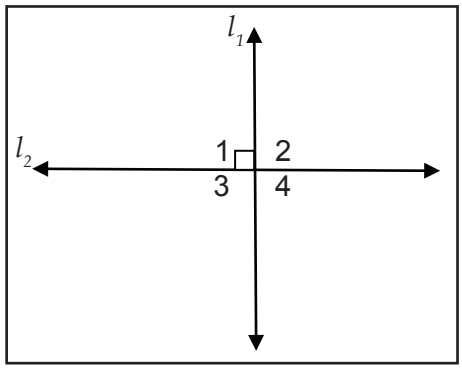
To prove that two lines are perpendicular, you must show that one of the following theorems is true:

1. If two lines are perpendicular to each other, then they form four right angles.



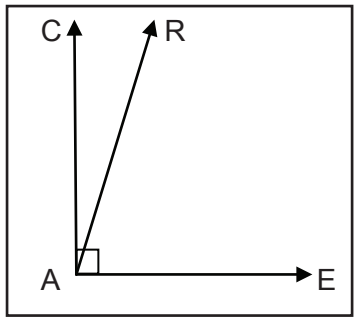


2. If the angles in a linear pair are congruent, then the lines containing their sides are perpendicular.



If $\angle 1$ and $\angle 2$ form a linear pair and $\angle 1 \cong \angle 2$, then $l_1 \perp l_2$.

3. If two angles are adjacent and complementary, the non-common sides are perpendicular.



If $\angle CAR$ and $\angle EAR$ are complementary and adjacent, then $\overline{AC} \perp \overline{AE}$.

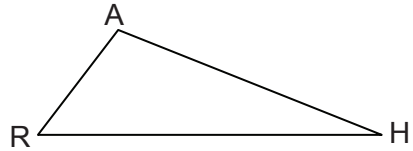
You may watch the video lesson using the given links. These videos will explain how to construct a perpendicular line to a point and a perpendicular line through a point not on a line.
http://www.youtube.com/watch?v=dK3S78SjPDw&feature=player_embedded

Activity 9 will test your skill and knowledge about perpendicular lines. This will prepare you also to understand the final task for this module. Come on. Try it!

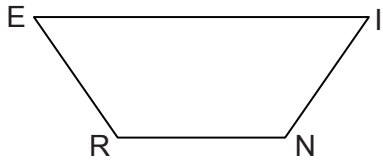
Activity 9 DRAW ME RIGHT!

Directions: Copy each figure in a separate sheet of bond paper. Draw the segment that is perpendicular from the given point to the identified side. Extend the sides if necessary.

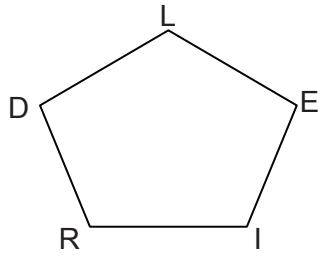
1. A to $\overline{RH}$



2. E to $\overline{RN}$



3. D to $\overline{IE}$



QUESTIONS ?

1. What did you use to draw the perpendicular segments?

   _____

   _____

2. How sure are you that the segments you drawn are really perpendicular to the indicated side?

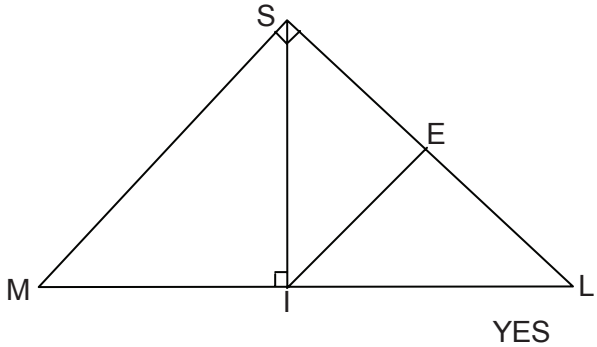
   _____

   _____

Activity 10 THINK TWICE!

Refer to the given figure and the given conditions in answering the succeeding questions. Raise your YES card if your answer is yes; otherwise, raise your NO card.

Given:
$\overline{MI} \cong \overline{IL}$
$\overline{SE} \cong \overline{EL}$
$m\angle SEI = 90$



	YES	NO
1. Is $\overline{ML} \perp \overline{IS}$?		
2. Is $\overline{MS} \perp \overline{SL}$?		
3. Is $SL \perp \overline{ML}$?		
4. Are $\angle MSI$ and $\angle ISL$ complementary angles?		

5. Are $\angle MIS$ and $\angle SIE$ complementary angles?
6. Is $\overline{IE}$ a perpendicular bisector of $\overline{SL}$?
7. Do $\angle MIS$ and $\angle SIL$ form a linear pair?
8. Is the $m\angle MIS$ = 90?
9. Is $\overline{SI}$ shorter than $\overline{SE}$?
10. Is $\overline{SE}$ shorter than $\overline{MI}$?

Activity 11 GENERALIZATION TABLE

Fill in the second, third, and fourth columns of the generalization table below by stating your present thoughts on the question.

"How can parallelism or perpendicularity of lines be established?"

	My Findings and Corrections	Supporting Evidence	Qualifying Conditions	

Discussion: ***KINDS OF QUADRILATERALS: A review***

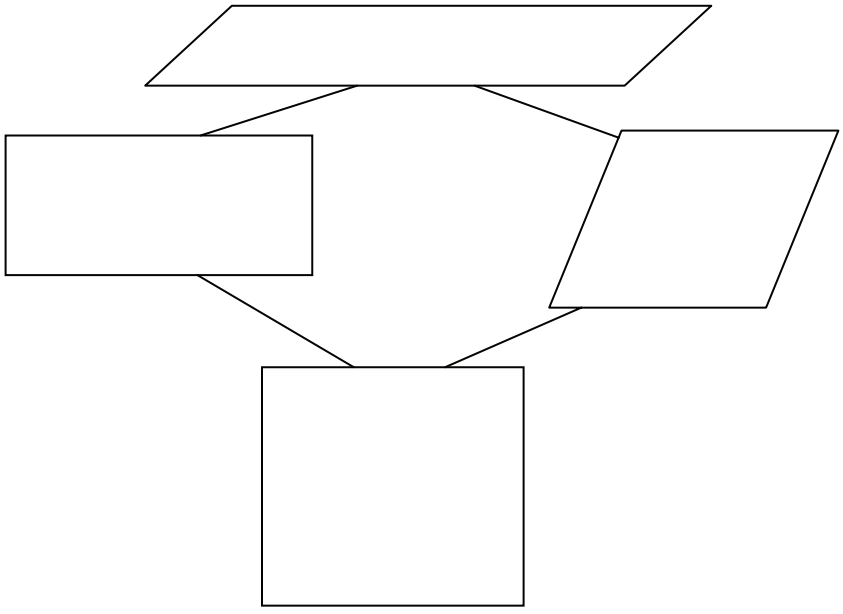
Quadrilateral is a polygon with four sides. The symbol ▭ is used in this module to indicate a quadrilateral. For example, ▭ ABCD, this is read as "quadrilateral ABCD".

Quadrilaterals are classified as follows:

1. Trapezium – a quadrilateral with no pair of parallel sides.
2. Trapezoid – a quadrilateral with exactly one pair of parallel sides. If the non-parallel sides are congruent, the trapezoid is said to be isosceles.
3. Parallelogram – a quadrilateral with two pairs of parallel and congruent sides. There are two special kinds of parallelogram: the rectangle which has four right angles and the rhombus which has four congruent sides. A square which has four congruent angles and four congruent sides can be a rectangle or a rhombus because it satisfies the definition for a rectangle and a rhombus.

Activity 12 SPECIAL QUADRILATERALS

Study the blank diagram below. Write the appropriate quadrilateral in the box. After which, complete the table below.



Direction: Place a check mark (√) in the boxes below if the quadrilateral listed along the top row has the properties listed in the left column.

Properties	Parallelogram	Rectangle	Rhombus	Square
Opposite sides are congruent.				
Opposite angles are congruent.				
Sum of the measures of the consecutive angles is 180°.				
Diagonals are congruent.				
Diagonals are perpendicular.				
Diagonals bisect each other				

QUESTIONS ?

1. What properties are common to rectangles, rhombi, and squares, if any?

   _____
   _____

2. What makes a rectangle different from a rhombus? A rectangle from a square? A rhombus from a square?

   _____
   _____

3. What do you think makes parallelograms special in relation to other quadrilaterals?

   _____
   _____

4. Are the properties of parallelograms helpful in establishing parallelism and perpendicularity of lines?

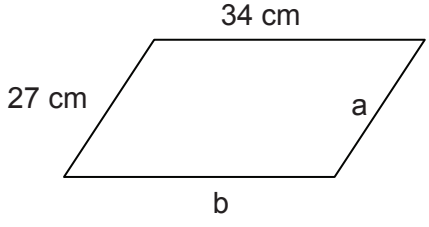
   _____
   _____

You may visit this URL to have more understanding of the properties of parallelogram.
http://www.youtube.com/watch?feature=player_detailpage&v=0rNjGNI1Uzo

Activity 13 HIDE AND SEEK!

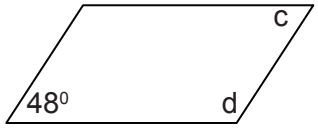
Each figure below is a parallelogram. Use your observations in the previous activity to find the value of the unknown parts.

1.

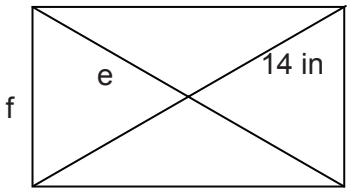


YOUR ANSWER

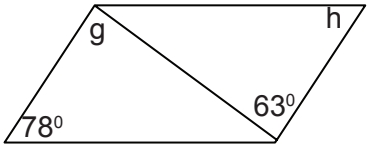
a = _____
b = _____



c = _____
d = _____

3. 

e = _____
f = _____

4. 

g = _____
h = _____

Discussion: ***Writing Proofs/Proving (A review)***

In the previous discussions, you have solved a lot of equations and inequalities by applying the different properties of equality and inequality. To name some, you have the APE (Addition Property of Equality), MPE (Multiplication Property of Equality) and TPE (Transitive Property of Equality). Now, you will use the same properties with some geometric definitions, postulates, and theorems to write a complete proof.

One of the tools used in proving is reasoning, specifically deductive reasoning. **Deductive reasoning** is a type of logical reasoning that uses accepted facts as reasons in a step-by-step manner until the desired statement is established.

A **proof** is a logical argument in which each statement you make is supported/justified by given information, definitions, axioms, postulates, theorems, and previously proven statements.

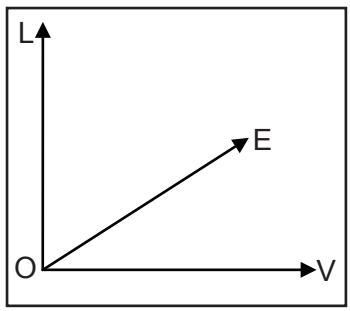
Proofs can be written in three different ways:

1. Paragraph Form/ Informal Proof:

The paragraph or informal proof is the type of proof where you write a paragraph to explain why a conjecture for a given situation is true.

Given: $\angle LOE$ and $\angle EOV$ are complementary

Prove: $\overline{LO} \perp \overline{OV}$



The Paragraph Proof:

Since $\angle LOE$ and $\angle EOV$ are complementary, then $m\angle LOE + m\angle EOV = 90$ by definition of complementary angles. Thus, $m\angle LOE + m\angle EOV = m\angle LOV$ by angle addition postulate and $m\angle LOV = 90$ by transitive property of equality. So, $\angle LOV$ is a right angle by definition of right angles. Therefore, $\overline{LO} \perp \overline{OV}$ by definition of perpendicularity.

2. Two-Column Form/ Formal Proof:

Two-column form is a proof with statements and reasons. The first column is for the statements and the other column for the reasons.

Using the same problem in #1, the proof is as follows:

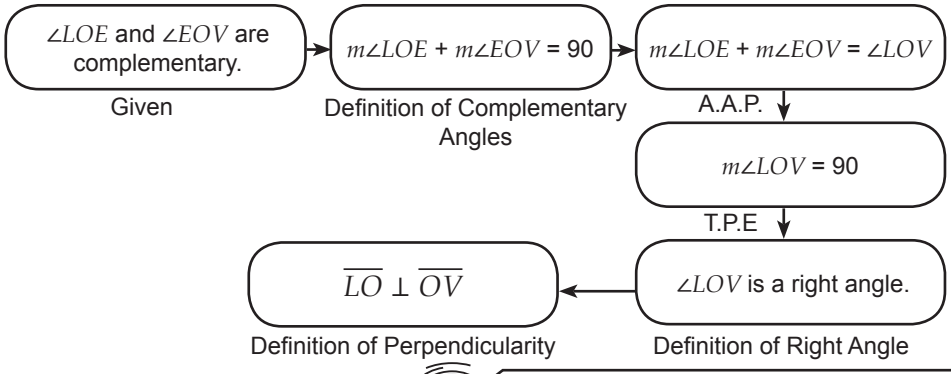
Statements	Reasons
1. $\angle LOE$ and $\angle EOV$ are complementary.	1. Given
2. $m\angle LOE + m\angle EOV = 90$	2. Definition of Complementary Angles
3. $m\angle LOE + m\angle EOV = m\angle LOV$	3. Angle Addition Postulate (AAP)
4. $m\angle LOV = 90$	4. Transitive Property of Equality (TPE)
5. $\angle LOV$ is a right angle.	5. Definition of Right Angle
6. $\overline{LO} \perp \overline{OV}$	6. Definition of Perpendicularity

You may watch the video lesson on this kind of proof using the following link: http://www.youtube.com/watch?feature=player_embedded&v=3Ti7-Ojr7Cg

3. Flowchart Form:

A flowchart-proof organizes a series of statements in a logical order, starting with the given statements. Each statement together with its justification is written in a box and arrows are used to show how each statement leads to another. It can make one's logic visible and help others follow the reasoning.

The flowchart proof of the problem in #1 can be done this way:



This URL shows you a video lessons in proving using flow chart. http://www.youtube.com/watch?feature=player_embedded&v=jgylP7yPgFY

The following rubric will be used in giving grades for writing proofs.

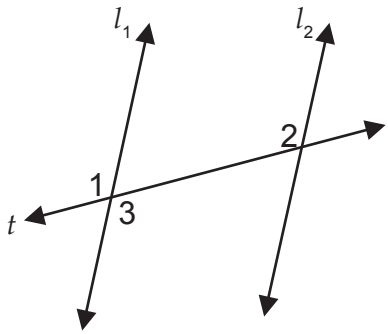
	4	3	2	1
Logic and Reasoning	The mathematical reasoning is sound and cohesive.	The mathematical reasoning is mostly sound, but lacking in some minor way.	The proof contains some flaws or omissions in mathematical reasoning.	The mathematical reasoning is either absent or seriously flawed. Use of mathematical terminology and notation
Use of mathematical terminology and notation	Notation is skillfully used; terminology is used flawlessly	Notation and terminology are used correctly with only a few exceptions.	There is a clear need for improvement in the use of terminology or notation	Terminology and notation are incorrectly and inconsistently used.
Correctness	The proof is complete and correct.	The proof is mostly correct, but has a minor flaw.	More than one correction is needed for a proper proof.	The argument given does not prove the desired result.

It's your turn. Accomplish Activity 14 and for sure you will enjoy!

Activity 14 COMPLETE ME!

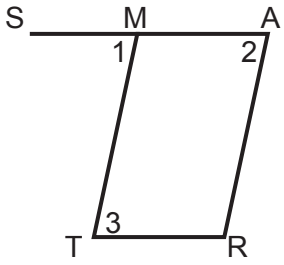
Complete each proof below:

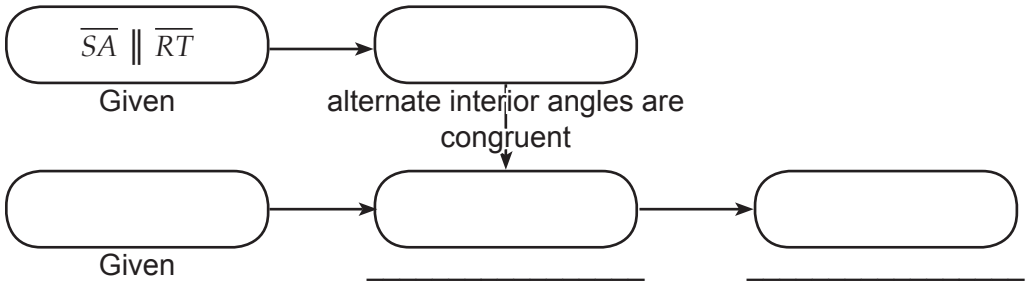
1. Given: Line t intersects l_1 and l_2 such that $\angle 1 \cong \angle 2$.
   Prove: $l_1 \parallel l_2$
   Proof:



Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. _____
2. _____	2. Vertical angles are congruent.
3. $\angle 3 \cong \angle 2$	3. Transitive Property of Congruence
4. $l_1 \parallel l_2$	4. _____

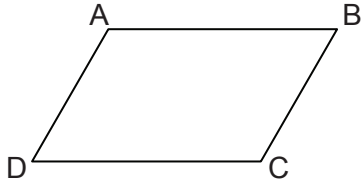
2. Given: $\overline{SA} \parallel \overline{RT}$
$\angle 2 \cong \angle 3$
Prove: $\overline{MT} \parallel \overline{AR}$
Proof:





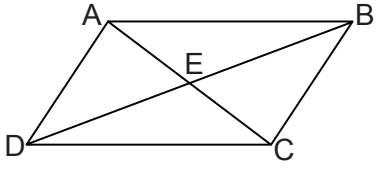
3. Given: $\square ABCD$ is a parallelogram.
Prove: $\angle A$ and $\angle B$ are supplementary.

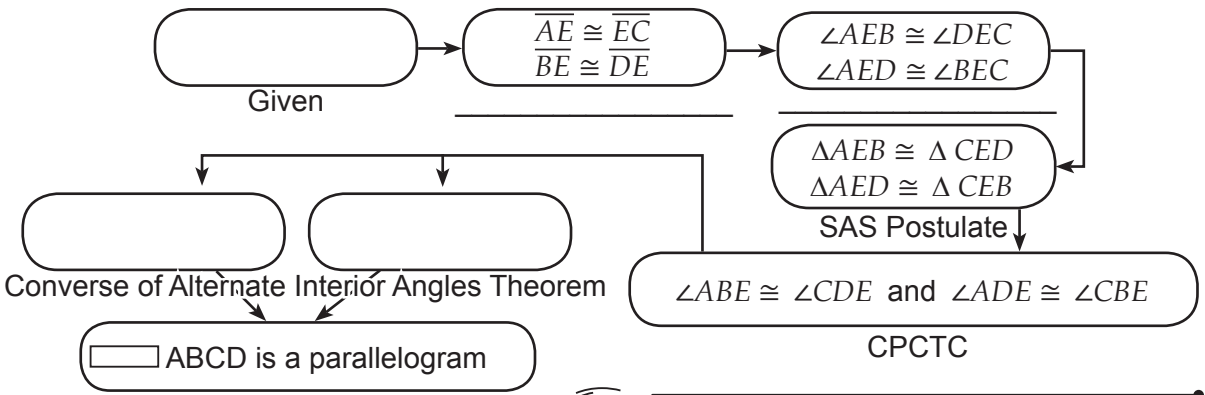
Proof:



Statements	Reasons
1. $\square$ABCD is a parallelogram.	1.
2. $\overline{BC} \parallel \overline{AD}$	2.
3. $\angle A$ and $\angle B$ are supplementary.	3.

4. Given: $\overline{AC}$ and $\overline{BD}$ bisect each other at E.
Prove: $\square$ABCD is a parallelogram.





In this section, the discussion was about the key concepts on parallelism and perpendicularity. Relationships of the different angle pairs formed by parallel lines cut by a transversal and the properties of parallelograms were also given emphasis. The different ways of proving through deductive reasoning were discussed with examples presented.

Go back to the previous section and compare your initial ideas with the discussion. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, go deeper by moving on to the next section.

What to Understand

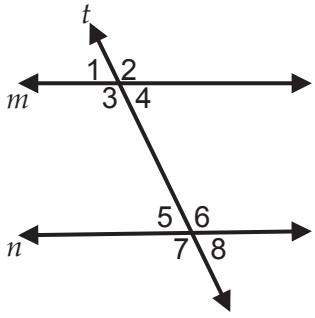
Your goal in this section is to take a closer look at some aspects of the topic. I hope that you are now ready to answer the exercises given in this section. Expectedly, the activities aim to intensify the application of the different concepts you have learned.

Activity 15 PROVE IT!

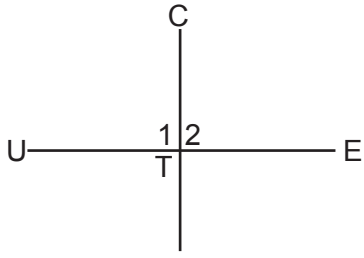
Prove the given statements below using any form of writing proofs.

1. Given:
   $m \parallel n$ and t is a transversal.

   Prove:
   $\angle 1$ and $\angle 7$ are supplementary.



2. In the figure, if $m\angle 1 = 3x + 15$, $m\angle 2 = 4x - 10$ prove that $\overline{CT}$ is perpendicular to $\overline{UE}$ if $x = 25°$.



QUESTIONS ?

1. What are the three different ways of proving deductively?

2. Which of the three ways is the best? Why?

3. How can one reason out deductively?

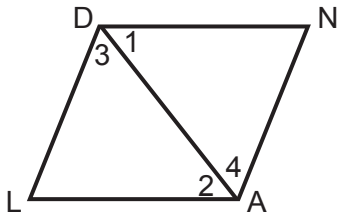
4. Why is there a need to study deductive reasoning? How is it related to real life? Cite a situation where deductive reasoning is applied.

Activity 16 PROVE SOME MORE... OKAY!

To strengthen your skill in proving deductively, provide a complete proof for each problem below. The use of flowchart is highly recommended.

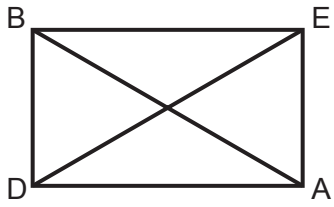
1. Given:
   ▭ LAND has $\overline{LA} \cong \overline{AN} \cong \overline{ND} \cong \overline{DL}$ with diagonal $\overline{AD}$.

   Prove: ▭ LAND is a rhombus.



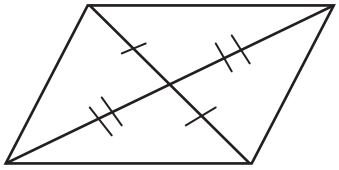
2. Given:
   ▭ BEAD is a rectangle.

   Prove: $\overline{AB} \cong \overline{DE}$



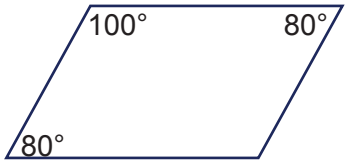
Activity 17 PARALLELOGRAMS

I. Study the markings on the given figures and shade ☺ if it is a parallelogram and ⊘ if it is not. If your answer is ☺ state the definition or theorem that justifies your answer.

1. 

☺ _____

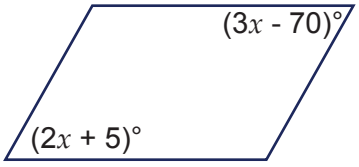
⊘ _____

2. 

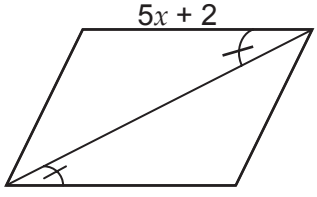
☺ _____

⊘ _____

II. What value of x will make each quadrilateral a parallelogram?

1. 

Solution:

2. 

Solution:

III. Show a complete proof:

Given: $\overline{CE} \parallel \overline{NI}$, $\overline{CE} \cong \overline{NI}$

Prove: ▭ NICE is a parallelogram.

Proof:

Activity 18 (REVISIT) AGREE OR DISAGREE!

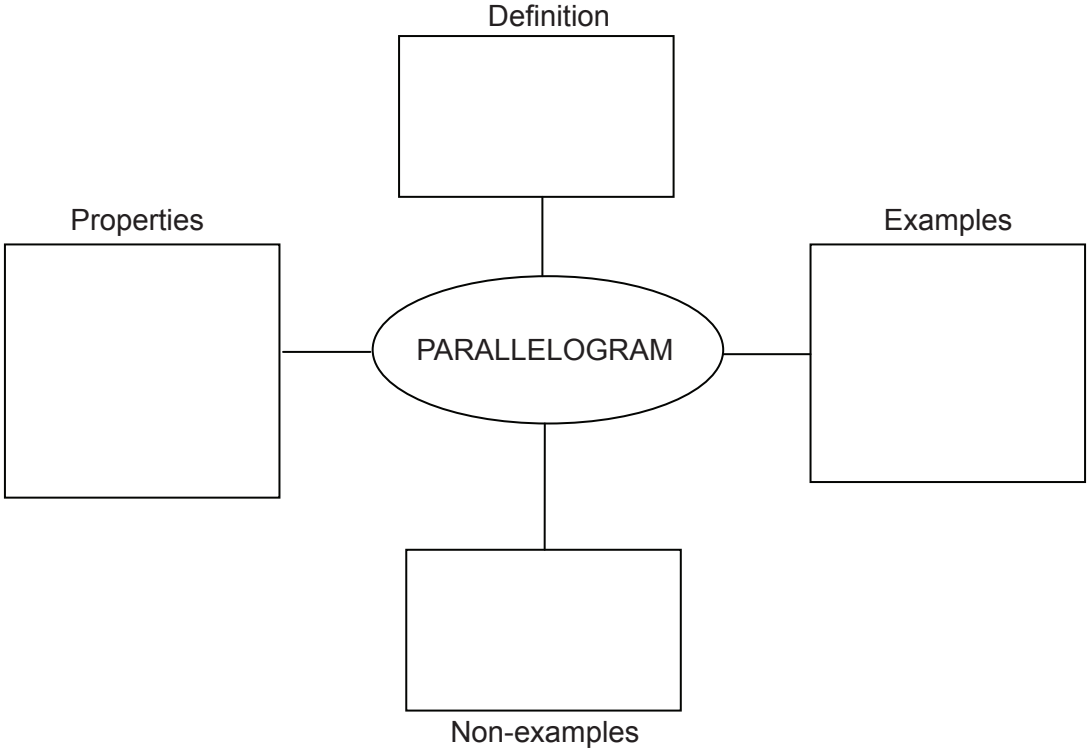
ANTICIPATION-REACTION GUIDE

Instruction: You were tasked to answer the first column during the earlier part of this module. Now, see how well you understood the lessons presented. Write A if you agree with the statement and write D if you disagree.

After-Lesson Response	TOPIC: Parallelism and Perpendicularity
	1. Lines that do not intersect are parallel lines.
	2. Skew lines are coplanar.
	3. Transversal lines are lines that intersects two or more lines.
	4. Perpendicular lines are intersecting lines.
	5. If two lines are parallel to a third line, then the two lines are parallel.
	6. If two lines are perpendicular to the same line, then the two lines are parallel.
	7. If one side of a quadrilateral is congruent to its opposite side, then the quadrilateral is a parallelogram.
	8. Diagonals of parallelogram bisect each other.
	9. Diagonals of parallelograms are congruent.
	10. Diagonals of parallelograms are perpendicular.
	11. Opposite sides of parallelograms are parallel.
	12. Opposite angles of a parallelogram are congruent.
	13. Consecutive angles of a parallelogram are congruent.
	14. Squares are rectangles.
	15. Squares are rhombi.

Activity 19 CONCEPT MAPPING

Group Activity: Summarize the important concepts about parallelograms by completing the concept map below. Present and discuss them in a large group.



Activity 20 GENERALIZATION TABLE

After a lot of exercises, it's now time for you to fill in the last column of the generalization table below by stating your conclusions or insights about parallelism and perpendicularity.

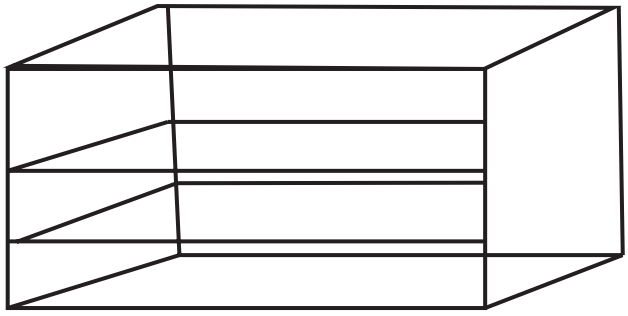
"How can parallelism or perpendicularity of lines be established?"

				My Generalizations

Activity 21 DESIGN IT!

You are working in a furniture shop as designer. One day, your immediate supervisor asked you to make a design of a wooden shoe rack for a new client, who is a well-known artist in the film industry. In as much as you don't want to disappoint your boss, you immediately think of the design and try to research on the different designs available on the internet.

Below is your design:



QUESTIONS ?

1. Based on your design, how will you ensure that the compartments of the shoe rack are parallel? Describe the different ways to ensure that the compartments are parallel.

2. Why is there a need to ensure parallelism on the compartments? What would happen if the compartments are not parallel?

3. How should the sides be positioned in relation to the base of the shoe rack? Does positioning of the sides in relation to the base matter?

Activity 22 SUMMATIVE TEST

The copy of the summative test will be given to you by your teacher. Do your best to answer all the items correctly. The result will be one of the bases of your grade.

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

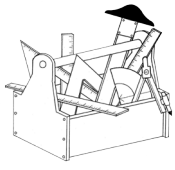
What to Transfer

Your goal in this section is to apply your learning to real-life situations. You will be given a practical task which will demonstrate your understanding.

This task challenges you to apply what you learned about parallel lines, perpendicular lines, parallelograms and the angles and segments related to these figures. Your work will be graded in accordance with the rubric presented.

Activity 23 DESIGNERS FORUM!

Scenario:



The Student Council of a school had a fund raising activity in order to put up a book case or shelf for the Student Council Office. You are a carpenter who is tasked to create a model of a book case/shelf using Euclidean tools (compass and a straight edge) and present it to the council adviser. Your output will be evaluated according to the following criteria: stability, accuracy, creativity and mathematical reasoning.

Goal – You are to create a model of a book case/shelf
Role – Carpenter
Audience – Council Adviser
Situation – The Student Council of a school had a fund raising activity in order to put up a book case or shelf for the Student Council Office.
Product – Book Case/Shelf
Standards – stability, accuracy, creativity, and mathematical reasoning.

RUBRIC FOR THE PERFORMANCE TASK

CRITERIA	Outstanding 4	Satisfactory 3	Developing 2	Beginning 1	RATING
Accuracy	The computations are accurate and show a wise use of the key concepts of parallelism and perpendicularity of lines.	The computations are accurate and show the use of key concepts of parallelism and perpendicularity of lines.	The computations are erroneous and show some use of the key concepts of, parallelism and perpendicularity of lines.	The computations are erroneous and do not show the use of key concepts of parallelism and perpendicularity of lines.	
Stability	The model is well fixed and in its place.	The model is firm and stationary.	The model is less firm and show slight movement.	The model is not firm and has the tendency to collapse.	
Creativity	The design is comprehensive and displays the aesthetic aspects of the mathematical concepts learned.	The design is presentable and makes use of the concepts of geometric representations.	The design makes use of the geometric representations but not presentable.	The design doesn't use geometric representations and not presentable.	
Mathematical Reasoning	The explanation is clear, exhaustive or thorough and coherent. It includes interesting facts and principles. It uses complex and refined mathematical reasoning.	The explanation is clear and coherent. It covers the important concepts. It uses effective mathematical reasoning.	The explanation is understandable but not logical. Some evidence of mathematical reasoning.	The explanation is incomplete and inconsistent. Little evidence of mathematical reasoning.	
				OVERALL RATING	

Activity 24 LESSON CLOSURE – REFLECTION ORGANIZER

You have accomplished the task successfully. This shows that you learned the important concepts in this module. To end this lesson meaningfully and to welcome you to the next module, I want you to accomplish this activity.

In this unit I learned about

_____
_____
_____

These concepts can be used in

_____
_____
_____

I understand that

_____
_____
_____

These are important because

_____
_____
_____

I can use the concepts of parallelism and perpendicularity in my life by

_____
_____
_____

In this section, your task was to create a model of a book case using protractor compass and a straight edge and present it to the council adviser.

How did you find the performance task? How did the task help you see the real-world application of the topic?

You have completed this lesson. Before you go to the next lesson, you have to answer the post assessment to evaluate your learning. Take time to answer the post assessment which will be given to you. If you do well, you may move on to the next module. If your score is not at the expected level, you have to go back and study the module again.

SUMMARY/SYNTHESIS/GENERALIZATION

In this module, you were given the opportunity to explore, learn, and apply the key concepts in parallelism and perpendicularity of lines. Doing the given activities and performing the transfer task with accuracy, creativity, stability, and use of mathematical reasoning were the evidence of your understanding of the lesson.

GLOSSARY OF TERMS USED IN THIS LESSON:

1. Adjacent Sides
   These are two non-collinear sides with a common endpoint.

2. Alternate Exterior Angles
   These are non-adjacent exterior angles that lie on opposite sides of the transversal.

3. Alternate Interior Angles
   These are non-adjacent interior angles that lie on opposite sides of the transversal.

4. Consecutive Angles
   These are two angles whose vertices are the endpoints of a common (included) side.

5. Consecutive Vertices
   These are the vertices which are at the endpoints of a side.

6. Corresponding Angles
   These are non-adjacent angles that lie on the same side of the transversal, one interior angle and one exterior angle.

7. Deductive Reasoning
   It is a type of logical reasoning that uses accepted facts to reason in a step-by-step manner until we arrive at the desired statement.

8. Flowchart-Proof
   It is a series of statements in a logical order, starting with the given statements. Each statement together with its reason written in a box, and arrows are used to show how each statement lead to another. It can make ones logic visible and help others follow the reasoning.

9. Kite
   It is a quadrilateral with two distinct pairs of adjacent congruent sides and no opposite sides congruent.

10. Opposite Angles
    These are two angles which do not have a common side.

11. Opposite Sides
    These are the two sides that do not have a common endpoint.

12. Paragraph or Informal Proof
    It is the type of proof where you write a paragraph to explain why a conjecture for a given situation is true.

13. Parallel lines
    Parallel lines are coplanar lines that do not intersect.

14. Parallelogram
    It is a quadrilateral with both pairs of sides parallel and congruent.

15. Perpendicular Bisector
    It is a line or a ray or another segment that is perpendicular to the segment and intersects the segment at its midpoint.

16. Perpendicular lines
    These are lines that intersect at 90^0- angle.

17. Proof
    It is a logical argument in which each statement you make is justified by a statement that is accepted as true.

18. Rectangle
    It is a parallelogram with four right angles.

19. Rhombus
    It is a parallelogram with four congruent sides.

20. Same-Side Interior Angles
    These are consecutive interior angles that lie on the same side of the transversal.

21. Same-Side Exterior Angles
    These are consecutive exterior angles that lie on the same side of the transversal.

22. Skew Lines
    Skew lines are non-coplanar lines that do not intersect.

23. Square
    It is a parallelogram with four congruent sides and four right angles.

24. Transversal
    It is a line that intersects two or more coplanar lines at different points.

25. Trapezoid
    It is a quadrilateral with exactly one pair of parallel sides.

26. Two-Column Form/Formal Proof
    It is the most formal proof with statements and reasons. The first column is for the statements and the other column for the reason.

POSTULATES OR THEOREMS ON PROVING LINES PARALLEL:

1. Given two coplanar lines cut by a transversal, if corresponding angles are congruent, then the two lines are parallel. (CACP)

2. Given two lines cut by a transversal, if alternate-interior angles are congruent, then the lines are parallel. (AICP)

3. If two lines are cut by a transversal such that the alternate-exterior angles are congruent, then the lines are parallel. (AECP)

4. Given two lines cut by a transversal, if same side interior angles are supplementary, then the lines are parallel. (SSIASP)

5. If two lines are cut by a transversal so that exterior angles on the same side of the transversal are supplementary, then the lines are parallel. (SSEASP)

6. In a plane, if two lines are both parallel to a third line, then they are parallel.

7. If two coplanar lines are perpendicular to a third line, then they are parallel to each other.

THEOREMS ON PROVING LINES PERPENDICULAR:

1. If two lines are perpendicular, then they form four right angles.

2. If the angles in a linear pair are congruent, then the lines containing their sides are perpendicular.

3. In a plane, through a point on a given line there is one and only one line perpendicular to the given line.

4. In a plane, a segment has a unique perpendicular bisector.

5. If two angles are adjacent and complementary, the non-common sides are perpendicular.

DEFINITIONS AND THEOREMS INVOLVING PARALLELOGRAMS

Given a parallelogram, related definition and theorems are stated as follows:

1. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
2. If a quadrilateral is a parallelogram, then 2 pairs of opposite sides are congruent.
3. If a quadrilateral is a parallelogram, then 2 pairs of opposite angles are congruent.
4. If a quadrilateral is a parallelogram, then the consecutive angles are supplementary.
5. If a quadrilateral is a parallelogram, then the diagonals bisect each other.
6. If a quadrilateral is a parallelogram, then the diagonals form two congruent triangles.

To prove a parallelogram, related definition and theorems are stated as follows: (Many of these theorems are converses of the previous theorems.)

1. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
2. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
3. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
4. If one angle is supplementary to both consecutive angles in a quadrilateral, then the quadrilateral is a parallelogram.
5. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
6. If one pair of opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram.

REFERENCES AND WEBSITE LINKS USED IN THIS LESSON:

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*http://oiangledlineswaves. jpg
Design by Becarry and Weblogs.com – Oct. 17, 2008
*http://brainden.com/images/cafe-wall.jpg
By Jan Adamovic
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These sites provide the optical illusions.

*http://www.mathwarehouse.com/geometry/angle/transveral-and-angles.php
*http://www.mathwarehouse.com/geometry/angle/interactive-transveral-angles.php
Created by Math Warehouse
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These sites provide exercises and review in the relationships of the different angles formed by parallel lines cut by a transversal.

*http://www.youtube.com/watch?v=AE3Pqhlvqw0&feature=related
*http://www.youtube.com/watch?v=VA92EWf9SRI&feature=relmfu
Created by Geometry4Everyone
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These sites provide an educational video presentation about parallel lines.

*http://www.nbisd.org/users/0006/docs/Textbooks/Geometry/geometrych3.pdf
By New Braunfels ISD
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This site provides reference to exercises involving parallel and perpendicular lines.

*http://www.regentsprep.org/Regents/math/geometry/GP8/PracParallel.htm
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Oswego City School District Regents Exam Prep Center
This site provides an interactive quiz which allows the students to practice solving problems on parallel lines cut by a transversal.

* http://www.nexuslearning.net/books/ml-geometry/
Created by McDougal Littell Geometry(2011)
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This site provides reference of the discussions and exercises involving parallel and perpendicular lines and quadrilaterals

*http://www.connect ED.mcgraw_hill.com
chapter_03_89527.3pdf
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This site provides lessons and exercises in Parallel and Perpendicular Lines.

*http://www.flvs.net/areas/studentservices/EOC/Documents/Geometry%20Practice%20Test%20with%20Answers.pdf
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2145 Metro Center Boulevard, Suite 200, Orlando, FL 32835
This site provides reference on the exercises involving quadrilaterals.

*http://www.cpm.org/pdfs/skillBuilders/GC/GC_Extra_Practice_Section12.pdf
Geometry Connections Extra Practice
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http://viking.coe.uh.edu/~jvanhook/geometry/chapter2/unit2lesson7notes.pdf
by University of Houston
Holt Geometry
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These sites provide reference and exercises in writing proofs.

*http://www.redmond.k12.or.us/14552011718214563/lib/14552011718214563/Lesson_4.7.pdf
Created by StudentWebLinks@Keymath.com Lesson 4.7
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This site provides discussions on how to make a flowchart and exercises in proving through deductive reasoning.

* http://www.regentsprep.org/Regents/math/geometry/GP9/LParallelogram.htm
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Oswego City School District Regents Exam Prep Center
This site provides discussions in the definitions and theorems involving parallelograms.

* http://www.glencoe.com/sec/teachingtoday/downloads/pdf/ReadingWritingMathClass. pdf
Author: Lois Edward
Mathematics Consultant
Minneapolis, Minnesota
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This site provides a reference of the concept map.