9

Mathematics

Learner's Material

Module 5: Quadrilaterals

This instructional material was collaboratively developed and reviewed by educators from public and private schools, colleges, and/or universities. We encourage teachers and other education stakeholders to email their feedback, comments, and recommendations to the Department of Education at action@deped.gov.ph.

We value your feedback and recommendations.

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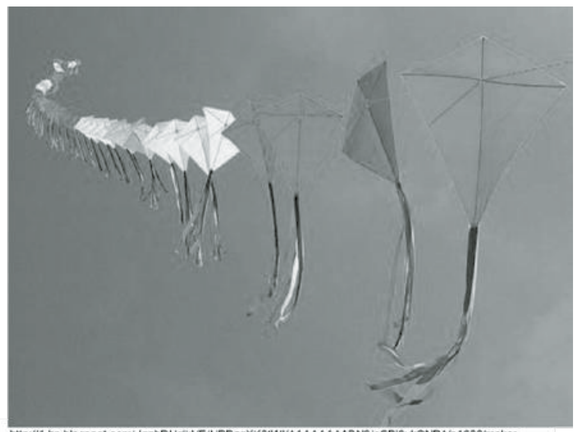
MODULE 5

Quadrilaterals

I. INTRODUCTION AND FOCUS QUESTIONS

Have you heard that the biggest dome in the world is found in the Philippines? Have you ever played billiards? Have you joined a kite-flying festival in your barangay? Have you seen a nipa hut made by Filipinos?

http://en.wikipedia.org/wiki/Philippine_Arena



http://1.bp.blogspot.com/-IqgbDUglkVE/UPDzqXK8tWI/AAAAAAAABN8/oSBi8ykQNPA/s1600/makar-sankrati-many-kites-Fresh+HD+Wallpapers.jpg

http://www.cuesportgroup.com/wp-content/uploads/2010/06/GameParty3_Wii_Billiards003.jpg

http://i1.treklens.com/photos/9392/img_0752.jpg

Study the pictures above. Look at the beautiful designs of the Philippine Arena, the lovely flying kites, the green billiard table and the nipa hut.

At the end of the module, you should be able to answer the following questions:

a. How can parallelograms be identified?
b. What are the conditions that guarantee a quadrilateral a parallelogram?
c. How do you solve problems involving parallelograms, trapezoids, and kites?
d. How useful are quadrilaterals in dealing with real-life situations?

II. LESSON AND COVERAGE

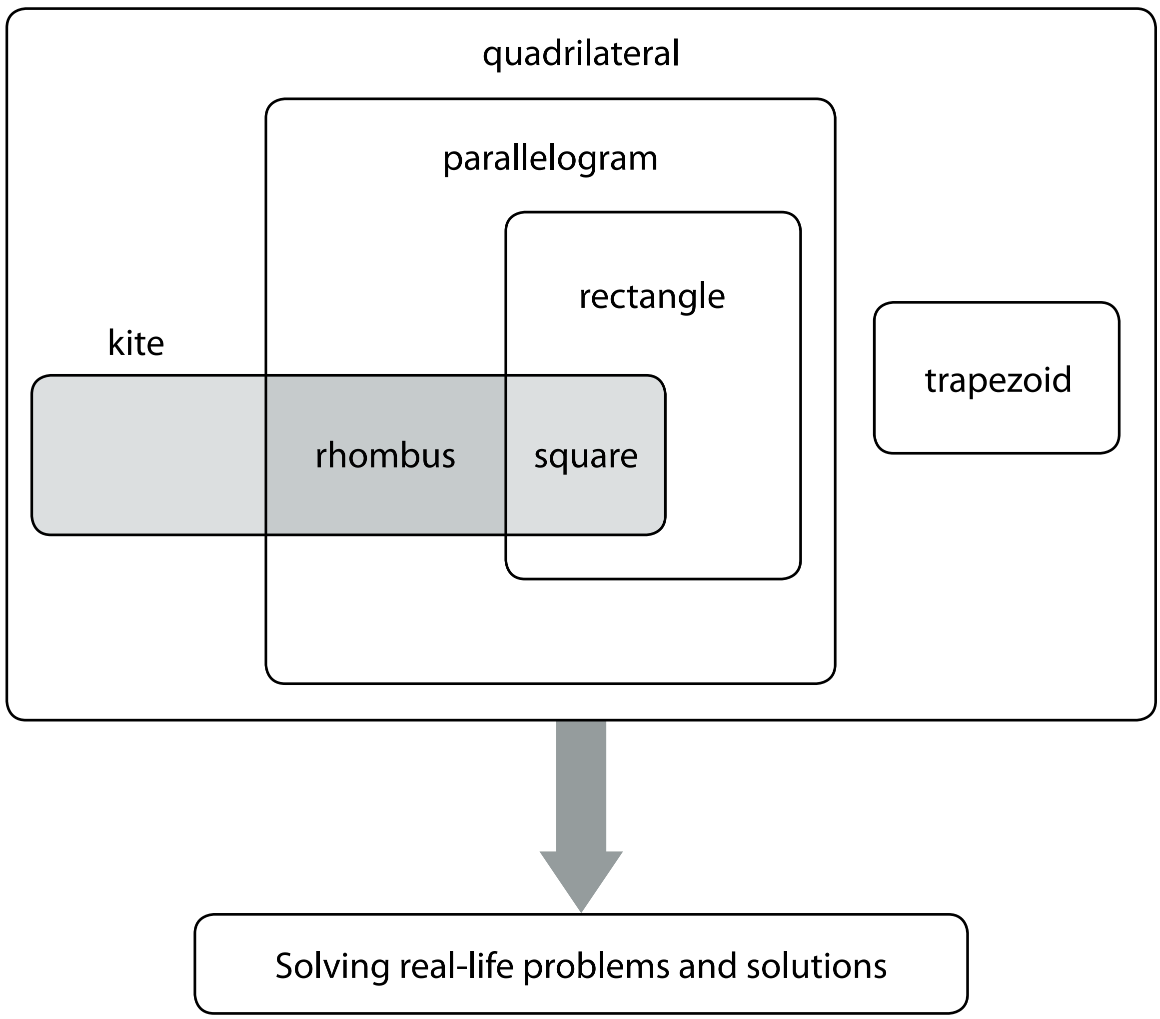
In this module, you will examine the aforementioned questions when you study the lesson on quadrilaterals:

In this lesson, you will learn to:

Competencies	• identify quadrilaterals that are parallelograms • determine the conditions that guarantee a quadrilateral a parallelogram
	• use properties to find measures of angles, sides and other quantities involving parallelograms
	• prove theorems on the different kinds of parallelogram (rectangle, rhombus, square)
	• prove the Midline Theorem • prove theorems on trapezoids and kites
	• solve problems involving parallelograms, trapezoids and kites

Module Map

Here is a simple map of what this entire module is all about.



III. PRE-ASSESSMENT

Part I

Find out how much you already know about this module. Write the letter of your answer, if your answer is not among the choices, write *e*. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. How do you describe any two opposite angles in a parallelogram?
   a. They are always congruent.
   b. They are supplementary.
   c. They are complementary.
   d. They are both right angles.

2. What can you say about any two consecutive angles in a parallelogram?
   a. They are always congruent.
   b. They are always supplementary.
   c. They are sometimes complementary.
   d. They are both right angles.

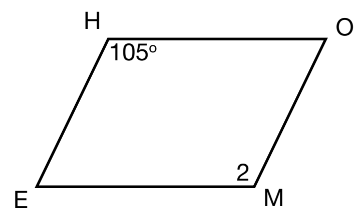
3. Which of the following statements is true?
   a. Every square is a rectangle.
   b. Every rectangle is a square.
   c. Every rhombus is a rectangle.
   d. Every parallelogram is a rhombus.

4. Which of the following statements could be false?
   a. The diagonals of a rectangle are congruent.
   b. The diagonals of an isosceles trapezoid are congruent.
   c. The diagonals of a square are perpendicular and bisect each other.
   d. The diagonals of a rhombus are congruent and perpendicular to each other.

5. Which of the following quadrilaterals has diagonals that do not bisect each other?
   a. Square
   b. Rhombus
   c. Rectangle
   d. Trapezoid

6. Which of the following conditions is not sufficient to prove that a quadrilateral is a parallelogram?
   a. Two pairs of sides are parallel.
   b. Two pairs of opposite sides are congruent.
   c. Two angles are supplementary.
   d. Two diagonals bisect each other.

7. What is the measure of $\angle 2$ in rhombus HOME?
   a. 75°
   b. 90°
   c. 105°
   d. 180°



8. Two consecutive angles of a parallelogram have measures $(x + 30)°$ and $[2(x - 30)]°$. What is the measure of the smaller angle?
   a. 30°
   b. 80°
   c. 100°
   d 140°

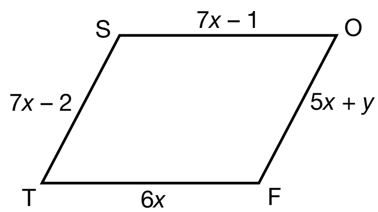
9. Which of the following statements is true?
   a. A trapezoid can have four equal sides.
   b. A trapezoid can have three right angles.
   c. The base angles of an isosceles trapezoid are congruent.
   d. The diagonals of an isosceles trapezoid bisect each other.

10. The diagonals of an isosceles trapezoid are represented by $4x - 47$ and $2x + 31$. What is the value of x?
    a. 37
    b. 39
    c. 107
    d. 109

11. A cross section of a water trough is in the shape of a trapezoid with bases measuring 2 m and 6 m. What is the length of the median of the trapezoid?
    a. 2 m
    b. 4 m
    c. 5 m
    d. 8 m

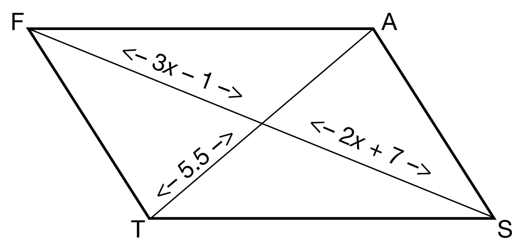
12. What are the measures of the sides of parallelogram SOFT in meters?

a. {2 m , 1 m}
b. {5 m , 6 m}
c. {8 m , 13 m}
d. {13 m , 15 m}



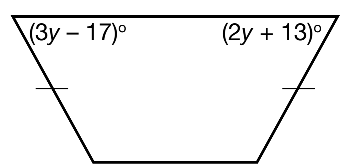
13. Find the length of the longer diagonal in parallelogram FAST.

a. 8
b. 31
c. 46
d. 52



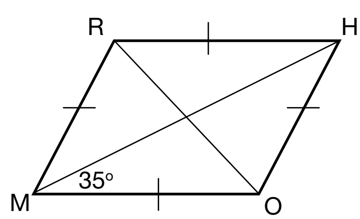
14. Find the value of y in the figure below.

a. 24
b. 30
c. 35
d. 50



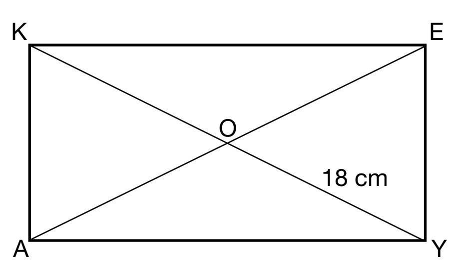
15. In rhombus RHOM, what is the measure of ∠ROH?

a. 35°
b. 45°
c. 55°
d. 90°



16. In rectangle KAYE, YO = 18 cm. Find the length of diagonal AE.

a. 6 cm
b. 9 cm
c. 18 cm
d. 36 cm



17. In quadrilateral RSTW, diagonals $\overline{RT}$ and $\overline{SW}$ are perpendicular bisectors of each other. Quadrilateral RSTW must be a:

    I. Rectangle II. Rhombus III. Square

    a. I
    b. II
    c. II and III
    d. I, II, and III

18. What condition will make parallelogram WXYZ a rectangle?
    a. $\overline{WX} \cong \overline{YZ}$
    b. $\overline{WX} \parallel \overline{YZ}$
    c. $\angle X$ is a right angle
    d. $\overline{WX}$ and $\overline{YZ}$ bisect each other

19. The perimeter of a parallelogram is 34 cm. If a diagonal is 1 cm less than its length and 8 cm more than its width, what are the dimensions of this parallelogram?
    a. 4 cm × 13 cm
    b. 5 cm × 12 cm
    c. 6 cm × 11 cm
    d. 7 cm × 10 cm

20. Which of the following statements is/are true about trapezoids?
    a. The diagonals are congruent.
    b. The median is parallel to the bases.
    c. Both a and b
    d. Neither a nor b

Part II

Read and understand the situation below then answer or perform what are asked.

Pepe, your classmate, who is also an SK Chairman in your Barangay Matayog, organized a KITE FLYING FESTIVAL. He informed your school principal to motivate students to join the said KITE FLYING FESTIVAL.

1. Suppose you are one of the students in your barangay, how will you prepare the design of the kite?
2 Make a design of the kite assigned to you.
3. Illustrate every part or portion of the kite including their measures.
4. Using the design of the kite made, determine all the mathematics concepts or principles involved.

Rubric			
Criteria	Poor (1 pt)	Fair (2 pts)	Good (3 pts)
Design	Design is basic, lacks originality and elaboration. Design is not detailed for construction.	Design is functional and has a pleasant visual appeal. Design includes most parts of a kite. Design lacks some details.	Design incorporates artistic elements and is original and well elaborated. Engineering design is well detailed for construction including four parts of a kite.
Planning	Overall planning is random and incomplete. Student is asked to return for more planning more than once.	Plan is perfunctory. It presents a basic design but is not well thought out. Contains little evidence of forward thinking or problem solving.	Plan is well thought out. Problems have been addressed prior to construction. Measurements are included. Materials are listed and gathered before construction. Student works cooperatively with adult leader and plans time well.
Construction	Work time is not used well. Construction is haphazard. Framing is loose. Covering is not even and tight. Not all components of a kite are present. Materials not used resourcefully.	Work time is not always focused. Construction is of fair quality. All components of a kite are present. Materials may not be used resourcefully.	Work time is focused. Construction is of excellent quality. All components of the kite are present. Care is taken to attach pieces carefully. Materials are used resourcefully. Student eagerly helps others when needed. Student works cooperatively with adult leader.

IV. LEARNING GOALS AND TARGETS

After going through this module, you should be able to demonstrate understanding of key concepts of quadrilaterals and be able to apply these to solve real-life problems. You will be able to formulate real-life problems involving quadrilaterals, and solve them through a variety of techniques with accuracy.

Quadrilaterals

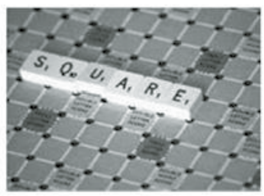
What to KNOW

This module shall focus on quadrilaterals that are parallelograms, properties of a parallelogram, theorems on the different kinds of parallelogram, the Midline theorem, theorems on trapezoids and kites, and problems involving parallelograms, trapezoids, and kites. Instill in mind the question *"How useful are the quadrilaterals in dealing with real-life situations?"* Let's start this module by doing Activity 1.

➤ Activity 1: Four-Sided Everywhere!

Study the illustrations below and answer the questions that follow.

http://farm1.staticflickr.com/146/357560359_bc9c8e4ad8_z.jpg



http://farm4.static.flickr.com/3485/329587332_f1353dc3cc_m.jpg

http://4.bp.blogspot.com/_bMI-KJUhzj4/TRAdodMgnKI/AAAAAAAACII/tZgZOS7Elw4/s1600/kite+(2).jpg

http://i.telegraph.co.uk/multimedia/archive/01809/satellite_1809335c.jpg

http://www.diarioartesgraficas.com/wp-content/uploads/2010/05/wood_ranch_rail_fence_21.jpg

http://library.thinkquest.org/28586/640x480x24/06_Trap/00_image.jpg

http://www.usa-traffic-signs.com/v/vspfiles/photos/Schzoadg_s-2.gif

http://www.dvbofficefurniture.co.uk/images/BB-10-FLXGT-Table.jpg

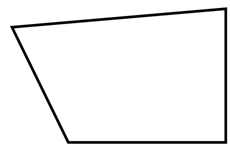
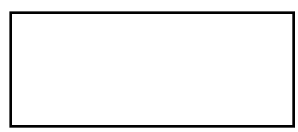
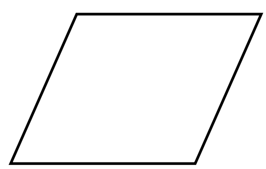
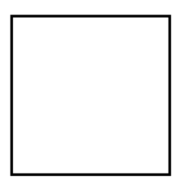
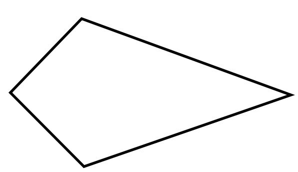
Questions:

1. What do you see in the illustrations above?
2. Do you see parts that show quadrilaterals?
3. Can you give some significance of their designs?
4. What might happen if you change their designs?
5. What are the different groups/sets of quadrilateral?

You have looked at the illustrations, determined the significance of their designs and some disadvantages that might happen in changing their designs, and classified the different groups/sets of quadrilateral. Now, you are going to refresh your mind on the definition of a quadrilateral and its kinds through the next activity.

➤ Activity 2: Refresh Your Mind!

Consider the table below. Given each figure, recall the definition of each quadrilateral and write it on your notebook.

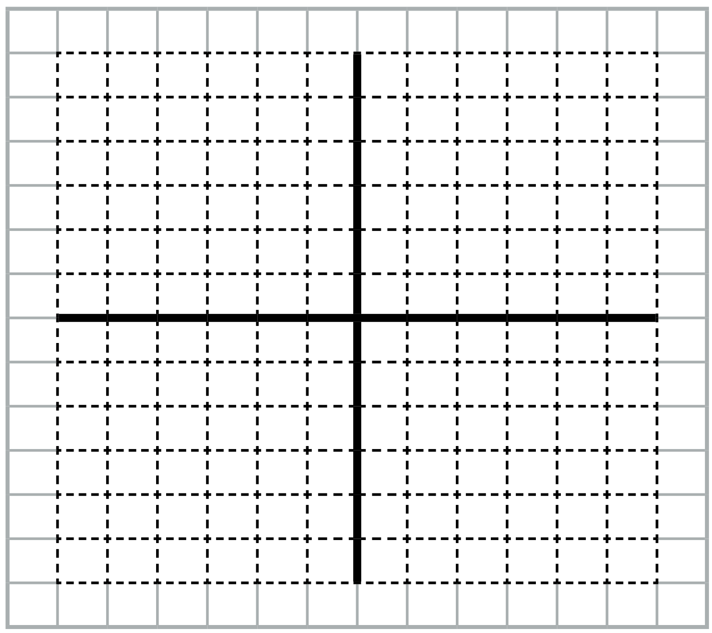
Kind	Figure	Definition
Quadrilateral		
Parallelogram		
Rectangle		
Rhombus		
Square		
Kite		

It feels good when refreshing some definitions taught to you before. This shall guide you in doing Activity 3 to determine which quadrilaterals are parallelograms.

➤ Activity 3: Plot, Connect, Identify

Plot the following sets of points in the Cartesian plane. Connect each given set of points consecutively to form a quadrilateral. Identify whether the figure is a parallelogram or not and answer the questions that follow.

1. (-1, 2) ; (-1, 0) ; (1, 0) ; (1, 2)
2. (1, 0) ; (3, 0) ; (0, -2) ; (3, -2)
3. (-4, -2) ; (-4, -4) ; (0, -2) ; (0, -4)
4. (3, 4) ; (2, 2) ; (3, 0) ; (4, 2)
5. (-4, 2) ; (-5, 1) ; (-3, 1) ; (-4, -2)
6. (-2, 4) ; (-4, 2) ; (-1, 2) ; (1, 4)



Questions:

1. Which among the figures are parallelograms? Why?
2. Which among the figures are not parallelograms? Why?

➤ Activity 4: Which Is Which?

Identify whether the following quadrilaterals are parallelograms or not. Put a check mark (✓) under the appropriate column and answer the questions that follow.

Quadrilateral	Parallelogram	Not Parallelogram
1. trapezoid		
2. rectangle		
3. rhombus		
4. square		

Questions:

1. Which of the quadrilaterals are parallelograms? Why?
2. Which of the quadrilaterals are not parallelograms? Why?

> You've just determined kinds of quadrilateral that are parallelograms. This time, you are ready to learn more about quadrilaterals that are parallelograms from a deeper perspective.

What to PROCESS

> You will learn in this section the conditions that guarantee that a quadrilateral is a parallelogram. After which, you will be able to determine the properties of a parallelogram and use these to find measures of angles, sides, and other quantities involving parallelograms. You are also going to prove the Midline Theorem and the theorems on trapezoids and kites. Keep in mind the question *"How useful are the quadrilaterals in dealing with real-life situations?"* Let us begin by doing *Check Your Guess 1* to determine your prior knowledge of the conditions that guarantee that a quadrilateral is a parallelogram.

✔ *Check Your Guess 1*

In the table that follows, write T in the second column if your guess on the statement is true; otherwise, write F. You are to revisit the same table later on and respond to your guesses by writing R if you were right or W if wrong under the third column.

Statement	My guess is...(T or F)	I was…(R or W)
1. In parallelogram ABCD, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$.		
2. If m∠F is 60°, then m∠G is also 60° in parallelogram EFGH.		
3. In parallelogram IJKL, $\overline{IK} \cong \overline{JL}$.		
4. $\overline{MO}$ and $\overline{NP}$ bisect each other in parallelogram MNOP.		
5. In parallelogram QRST, $\overline{RT}$ divides it into two congruent triangles.		

Quadrilaterals That Are Parallelograms

➤ Activity 5: Fantastic Four!

Form a group of four members and require each member to have the materials needed. Follow the given procedures below and answer the questions that follow.

Materials: protractor, graphing paper, ruler, pencil, and compass

Procedures:

1. Each member of the group shall draw a parallelogram on a graphing paper. (parallelogram OBEY, rectangle GIVE, rhombus THNX, and square LOVE)
2. Measure the sides and the angles, and record your findings in your own table similar to what is shown below.
3. Draw the diagonals and measure the segments formed by the intersecting diagonals. Record your findings in the table.
4. After answering the questions, compare your findings with your classmates.

In your drawing, identify the following:		Measurement	Are the measurements equal or not equal?
pairs of opposite sides			
pairs of opposite angles			
pairs of consecutive angles			
pairs of segments formed by intersecting diagonals			

Questions:

1. Based on the table above, what is true about the following?
   a. pairs of opposite sides
   b. pairs of opposite angles
   c. pairs of consecutive angles
   d. pairs of segments formed by intersecting diagonals
2. What does each diagonal do to a parallelogram?

3. Make a conjecture about the two triangles formed when a diagonal of a parallelogram is drawn. Explain your answer.
4. What can you say about your findings with those of your classmates?
5. Do the findings apply to all kinds of parallelogram? Why?

Your answers to the questions show the conditions that guarantee that a quadrilateral is a parallelogram. As a summary, complete the statements that follow using the correct words/phrases based on your findings.

In this section, you shall prove the different properties of a parallelogram. These are the following:

Properties of Parallelogram

1. In a parallelogram, any two opposite sides are congruent.
2. In a parallelogram, any two opposite angles are congruent.
3. In a parallelogram, any two consecutive angles are supplementary.
4. The diagonals of a parallelogram bisect each other.
5. A diagonal of a parallelogram forms two congruent triangles.

You must remember what you have learned in proving congruent triangles. Before doing the different *Show Me!* series of activities, check your readiness by doing *Check Your Guess 2* that follows.

✔ *Check Your Guess 2*

In the table that follows, write T in the second column if your guess on the statement is true; otherwise, write F. You are to revisit the same table later on and respond to your guesses by writing R if you were right or W if wrong under the third column.

Statement	My guess is... (T or F)	I was… (R or W)
1. A quadrilateral is a parallelogram if both pairs of opposite sides are parallel.		
2. A quadrilateral is a parallelogram if both pairs of opposite sides are congruent.		
3. A quadrilateral is a parallelogram if both pairs of opposite angles are congruent.		
4. A quadrilateral is a parallelogram if any two consecutive angles are complementary.		
5. A quadrilateral is a parallelogram if exactly one pair of adjacent sides is perpendicular.		
6. A quadrilateral is a parallelogram if one pair of opposite sides are both congruent and parallel.		

➤ Activity 6.1: Draw Me!

Using a straightedge, compass, protractor, and pencil, construct the quadrilaterals, given the following conditions.

1. Quadrilateral ABCD so that $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$.

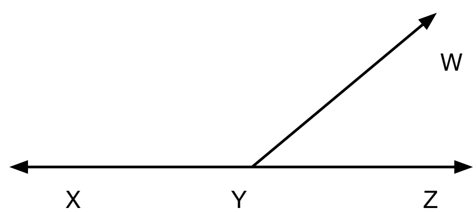
   Steps:

   a. Draw ∠DAB.

   b. Locate C so that $\overline{DC} \cong \overline{AB}$ and $\overline{CB} \cong \overline{DA}$.

   Hint: From D, strike an arc with radius AB.
   From B, strike an arc with radius DA.

2. In the figure below, $\angle XYW$ and $\angle ZYW$ form a linear pair.



   Draw quadrilateral EFGH so that:

   $\angle H \cong \angle WYZ$; $\angle G \cong \angle XYW$

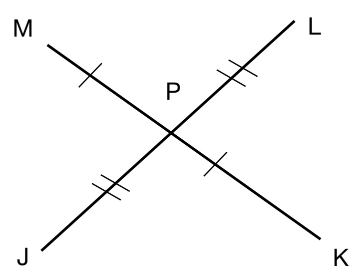
   $\angle F \cong \angle WYZ$; $\angle E \cong \angle XYW$

3. Diagonals $\overline{JL}$ and $\overline{MK}$ bisect each other.

   Steps:

   a. Draw $\overline{JL}$, locate its midpoint P.

   b. Draw another line $\overline{MK}$ passing through P so that $\overline{MP} \cong \overline{KP}$.



   c. Form the quadrilateral MJKL.

4. A diagonal bisects the quadrilateral into two congruent triangles.

   Steps:

   a. Draw ΔABC.

b. Construct $\overline{AD}$ and $\overline{BD}$ so that $\overline{AD} \cong \overline{BC}$ and $\overline{BD} \cong \overline{AC}$.
c. Construct quadrilateral ACBD.
(Note: What do we call $\overline{AB}$ in relation to quadrilateral ACBD?)

5. One pair of opposite sides are both congruent and parallel.
Steps:
a. Draw segment $\overline{AB}$.
b. From an external point C, draw a $\overline{CQ} \parallel \overline{AB}$.
c. On $\overline{CQ}$, locate point D so that $\overline{CD} \cong \overline{AB}$.
d. Form the quadrilateral ABDC.

Question:

What quadrilaterals have you formed in your constructions 1–5?

Conditions that Guarantee that a Quadrilateral a Parallelogram

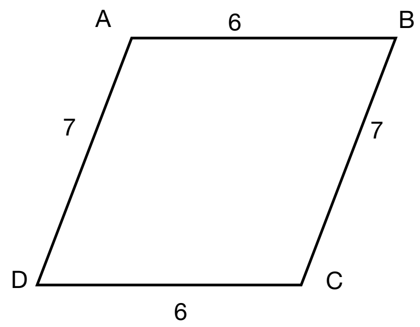
1. A quadrilateral is a parallelogram if both pairs of ________________ sides are ________________.
2. A quadrilateral is a parallelogram if both pairs of ________________ angles are ________________.
3. A quadrilateral is a parallelogram if both pairs of ________________ angles are ________________.
4. A quadrilateral is a parallelogram if the ____________________ bisect each other.
5. A quadrilateral is a parallelogram if each __________________ divides a parallelogram into two _________________________________.
6. A quadrilateral is a parallelogram if one pair of opposite sides are both ______________ and ______________.

You've just determined the conditions that guarantee that a quadrilateral is a parallelogram. Always bear in mind those conditions to help and guide you as you go on. Do the following activities and apply the above conditions.

➤ Activity 6.2: Defense! Defense!

Study the following parallelograms below and answer the questions given below each figure.

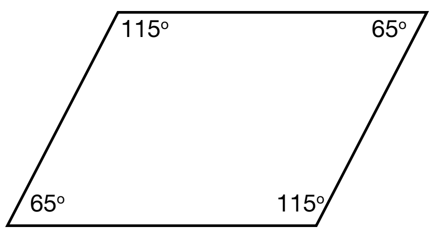
1.



Questions:

- What condition guarantees that the figure is a parallelogram?
- Why did you say so?

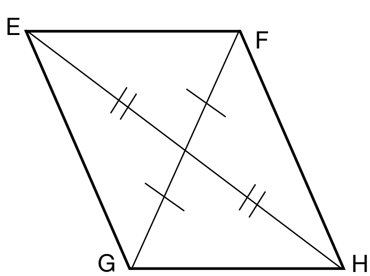
2.



Questions:

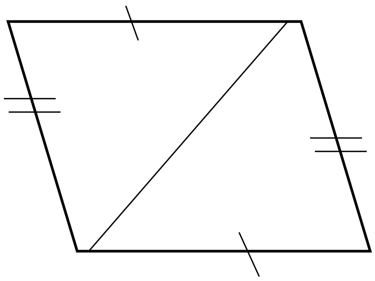
- What condition/s guarantee/s that the figure is a parallelogram?
- Why did you say so?

3.



Questions:

- What condition guarantees that the figure is a parallelogram?
- Why?

4. 

Questions:

- What condition guarantees that the figure is a parallelogram?
- Why?

Your observations in previous activities can be proven deductively using the two-column proof. But before that, revisit *Check Your Guess 1* and see if your guesses were right or wrong. How many did you guess correctly?

Properties of Parallelogram

Parallelogram Property 1

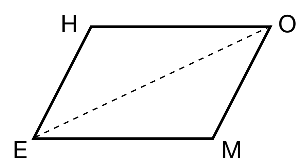
In a parallelogram, any two opposite sides are congruent.

Show Me!

Given: Parallelogram HOME

Prove: $\overline{HO} \cong \overline{ME}$; $\overline{OM} \cong \overline{HE}$

Proof:



Statements	Reasons
1.	1. Given
2.	2. Definition of a parallelogram
3. Draw $\overline{EO}$	3.
4.	4. Alternate Interior Angles Are Congruent (AIAC).
5.	5. Reflexive Property
6. ΔHOE ≅ ΔMEO	6.
7. $\overline{HO} \cong \overline{ME}$; $\overline{OM} \cong \overline{HE}$	7.

You've just proven a property that any two opposite sides of a parallelogram are congruent. Remember that properties already proven true shall be very useful as you go on. Now, do the next *Show me!* activity to prove another property of a parallelogram.

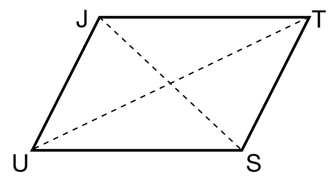
Parallelogram Property 2

In a parallelogram, any two opposite angles are congruent.

Show Me!

Given: Parallelogram JUST

Prove: ∠JUS ≅ ∠STJ; ∠UJT ≅ ∠TSU



Proof:

Statements	Reasons
1.	1. Given
2. Draw $\overline{UT}$ and $\overline{JS}$.	2.
3.	3. Parallelogram Property 1
4.	4. Reflexive Property
5. ΔTUJ ≅ ΔUTS; ΔSTJ ≅ ΔJUS	5.
6. ∠JUS ≅ ∠STJ; ∠UJT ≅ ∠TSU	6.

You've just proven another property that any two opposite angles of a parallelogram are congruent. Now, proceed to the next *Show Me!* activity to prove the third property of a parallelogram.

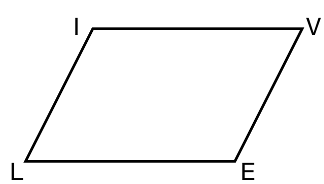
Parallelogram Property 3

In a parallelogram, any two consecutive angles are supplementary.

Show Me!

Given: Parallelogram LIVE

Prove: ∠I and ∠V are supplementary.
∠V and ∠E are supplementary.
∠E and ∠L are supplementary.
∠L and ∠I are supplementary.



Proof:

Statements	Reasons
1.	1. Given
2. $\overline{LI} \parallel \overline{VE}$	2.
3. ∠I and ∠V are supplementary.	3.

4. $\angle I \cong \angle E$; $\angle V \cong \angle L$	4.
5.	5. An angle that is supplementary to one of two congruent angles is supplementary to the other also.

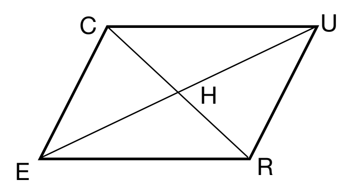
Note: The proof that other consecutive angles are supplementary is left as an exercise.

You are doing great! This time, do the next *Show Me!* activity to complete the proof of the fourth property of a parallelogram.

Parallelogram Property 4

The diagonals of a parallelogram bisect each other.

Show Me!



Given: Parallelogram CURE with diagonals $\overline{CR}$ and $\overline{UE}$

Prove: $\overline{CR}$ and $\overline{UE}$ bisect each other.

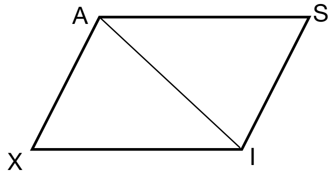
Proof:

Statements	Reasons
1.	1. Given
2. $\overline{CR} \cong \overline{UE}$	2.
3. $\overline{CR} \parallel \overline{UE}$	3.
4. $\angle CUE \cong \angle REU$;	4.
5. $\angle CHU \cong \angle RHE$	5.
6.	6. SAA Congruence Postulate
7. $\overline{CH} \cong \overline{RH}$; $\overline{EH} \cong \overline{UH}$	7.
8. $\overline{CR}$ and $\overline{UE}$ bisect each other.	8.

To determine the proof of the last property of a parallelogram, do the next *Show Me!* activity that follows.

Parallelogram Property 5

A diagonal of a parallelogram divides the parallelogram into two congruent triangles.



Show Me!

Given: Parallelogram AXIS with diagonal $\overline{AI}$

Prove: ΔAXI ≅ ΔISA

Proof:

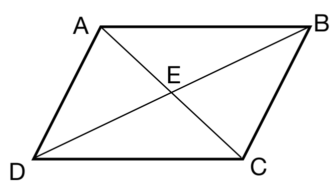
Statements	Reasons
1.	1. Given
2. $\overline{AX} \parallel \overline{IS}$ and $\overline{AS} \parallel \overline{IX}$	2.
3. ∠XAI ≅ ∠SIA	3.
4.	4. Reflexive Property
5. ∠XIA ≅ ∠SAI	5.
6. ΔAXI ≅ ΔISA	6.

You are now ready to use the properties to find the measures of the angles, sides, and other quantities involving parallelograms. Consider the prepared activity that follows.

Solving Problems on Properties of Parallelogram

➤ Activity 7: Yes You Can!

Below is parallelogram ABCD. Consider each given information and answer the questions that follow.



1. Given: AB = $(3x - 5)$ cm, BC = $(2y - 7)$ cm, CD =$(x + 7)$ cm and AD = $(y + 3)$ cm.
   a. What is the value of x?
   b. How long is $\overline{AB}$?
   c. What is the value of y?
   d. How long is $\overline{AD}$?
   e. What is the perimeter of parallelogram ABCD?

Questions:

- How did you solve for the values of x and y?
- What property did you apply to determine the lengths of $\overline{AB}$ and $\overline{AD}$?

2. $\angle$BAD measures $(2a + 25)°$ while $\angle$BCD measures $(3a - 15)°$.
   a. What is the value of a?
   b. What is m $\angle$BAD?
   c. What is m $\angle$CBA?

   Questions:
   - How did you find the value of a?
   - What property did you apply to solve for m $\angle$CBA?

3. Diagonals AC and BD meet at E. DE is 8 cm and AC is 13 cm.
   a. How long is BD?
   b. How long is AE?

   Questions:
   - How did you solve for the lengths of $\overline{BD}$ and $\overline{AE}$?
   - What property did you apply?

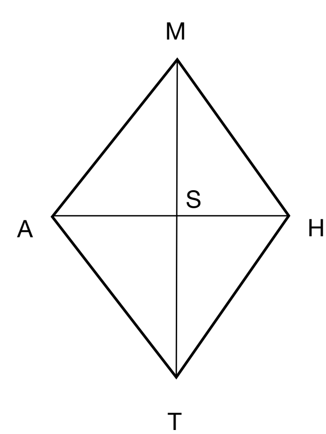
> You should always remember what you have learned in the past. It pays best to instill in mind what had been taught. Now, prepare for a quiz.

QUIZ 1

A. Refer to the given figure at the right and answer the following.

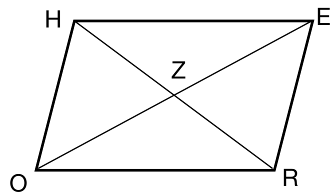
Given: $\square$MATH is a parallelogram.

1. $\overline{MA} \cong$ ______
2. $\Delta\overline{MAH} \cong$ ______
3. $\overline{MS} \cong$ ______
4. ΔTHM $\cong$ ______
5. $\angle$ATH $\cong$ ______
6. If m $\angle$MHT = 100, then m $\angle$MAT ______
7. If m $\angle$AMH = 100, then m $\angle$MHT ______
8. If MH = 7, then AT = ______
9. If AS = 3, then AH = ______
10. If MT = 9, then SM = _______



B. Answer the following.

1. Given: HE = $2x$
   OR = $x + 5$
   Find: HE
2. Given: m $\angle$HER = $5y - 26$
   m $\angle$ROH = $2y - 40$
   Find: m $\angle$ROH
3. Given: m $\angle$OHE = 3 m $\angle$HER
   Find: m $\angle$OHE and m $\angle$HER
4. Given: HZ = $4a - 5$
   RZ = $3a + 5$
   Find: HZ
5. Given: OZ = $12b + 1$
   ZE = $2b + 21$
   Find: ZE



After applying the different properties of a parallelogram, you are now ready to prove theorems on the different kinds of parallelogram.
But before that, revisit *Check Your Guess 2* and see if your guesses were right or wrong. How many did you guess correctly?

What are the kinds of parallelogram? What are the different theorems that justify each kind? Let's discover the theorems on the different kinds of quadrilateral by doing first *Check Your Guess 3* that follows.

✔ *Check Your Guess 3*

In the table that follows, write AT in the second column if you guess that the statement is always true, ST if it's sometimes true, and NT if it is never true. You are to revisit the same table later and respond to your guesses by writing R if you were right or W if wrong under the third column.

Statement	My guess is… (AT, ST or NT)	I was… (R or W)
1. A rectangle is a parallelogram.		
2. A rhombus is a square.		
3. A parallelogram is a rectangle.		

4. A rhombus is a parallelogram.		
5. A rectangle is a rhombus.		
6. A square is a rhombus.		
7. A rhombus is a rectangle.		
8. A parallelogram is a rhombus.		
9. A square is a parallelogram.		
10. A square is a rectangle.		

Theorems on Rectangle

➤ Activity 8: I Wanna Know!

Do the procedures below and answer the questions that follow.

Materials Needed: bond paper, protractor, ruler, pencil, and compass

Procedure:

1. Mark two points O and P that are 10 cm apart.
2. Draw parallel segments from O and P which are 6 cm each, on the same side of $\overline{OP}$ and are perpendicular to $\overline{OP}$.
3. Name the endpoints from O and P as H and E, respectively, and draw $\overline{HE}$.
4. Draw the diagonals of the figure formed.

Questions:

1. Measure ∠OHE and ∠PEH. What did you find?
2. What can you say about the four angles of the figure?
3. Measure the diagonals. What did you find?
4. Does quadrilateral HOPE appear to be a parallelogram? Why?
5. What specific parallelogram does it represent?

Activity 8 helped you discover the following theorems related to rectangles:

- Theorem 1. If a parallelogram has one right angle, then it has four right angles and the parallelogram is a rectangle.
- Theorem 2. The diagonals of a rectangle are congruent.

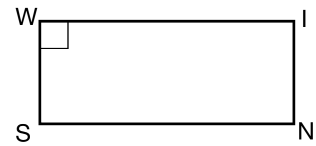
Just like what you did to the properties of a parallelogram, you are going to prove the theorems on rectangles above. Prove Theorem 1 by doing the *Show Me!* activity that follows.

Theorem 1. *If a parallelogram has a right angle, then it has four right angles and the parallelogram is a rectangle.*

Show Me!

Given: ▱WINS is a parallelogram with
∠W is a right angle.

Prove: ∠I, ∠N, and ∠S are right angles.



Proof:

Statements	Reasons
1.	1. Given
2. ∠W = 90 m	2.
3.	3. In a parallelogram, opposite angles are congruent.
4. m ∠W = m ∠N m ∠I = m ∠S	4.
5. m ∠N = 90 m	5.
6. m ∠W + m ∠I = 180	6.
7. 90 + m ∠I = 180	7.
8.	8. Reflexive Property
9. m ∠I = 90	9.
10.	10. Substitution (SN 4 and 9)
11. ∠I, ∠N, and ∠S are right angles.	11.
12.	12. Definition of rectangle.

Note: SN: Statement Number

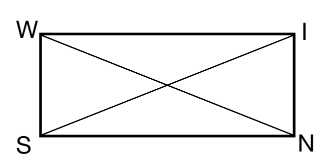
Congratulations! You contributed much in proving Theorem 1. Now, you are ready to prove Theorem 2.

Theorem 2. *The diagonals of a rectangle are congruent.*

Show Me!

Given: ▱WINS is a rectangle with diagonals
$\overline{WN}$ and $\overline{SI}$.

Prove: $\overline{WN} \cong \overline{SI}$



Statements	Reasons
1.	1. Given
2. $\overline{WS} \cong \overline{IN}$	2.
3. ∠WSN and ∠INSare right angles.	3.
4.	4. All right angles are congruent.
5. $\overline{SN} \cong \overline{NS}$	5.
6.	6. SAS Congruence Postulate
7. $\overline{WN} \cong \overline{IS}$	7.

Amazing! Now, let's proceed to the next kind of parallelogram by doing Activity 9.

Theorems on Rhombus

➤ Activity 9: I Wanna Know More!

Do the procedures below and answer the questions that follow.

Materials: bond paper, protractor, pencil, and ruler

Procedure:

1. Draw a rhombus that is not necessarily a square. Since a rhombus is also a parallelogram, you may use a protractor to draw your rhombus. Name the rhombus NICE. (Note: Clarify how a rhombus can be drawn based on its definition, parallelogram all of whose sides are congruent.)
2. Draw diagonals $\overline{NC}$ and $\overline{IE}$ intersecting at R.
3. Use a protractor to measure the angles given in the table below.

Angle	∠NIC	∠NIE	∠INE	∠INC	∠NRE	∠CRE
Measure						

Questions:

1. Compare the measures of ∠NIC and ∠NIE. What did you observe?
2. What does $\overline{IE}$ do to ∠NIC? Why?
3. Compare the measures of ∠INE and ∠INC. What did you observe?
4. What does $\overline{NC}$ do to ∠INE? Why?
5. Compare the measures of ∠NRE and ∠CRE. What did you observe?

6. What angle pair do ∠NRE and ∠CRE form? Why?
7. How are the diagonals $\overline{NC}$ and $\overline{IE}$ related to each other?

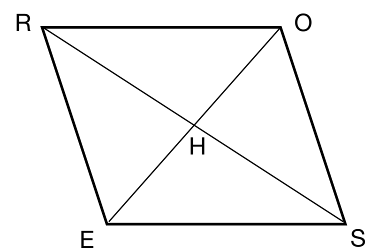
> Activity 9 led you to the following theorems related to rhombus:
> - Theorem 3. The diagonals of a rhombus are perpendicular.
> - Theorem 4. Each diagonal of a rhombus bisects opposite angles.
>
> To prove the theorems above, do the succeeding *Show Me!* activities.

Theorem 3. *The diagonals of a rhombus are perpendicular.*

Show Me!

Given: Rhombus ROSE

Prove: $\overline{RS} \perp \overline{OE}$



Proof:

Statements	Reasons
1.	1. Given
2. $\overline{OS} \cong \overline{RO}$	2.
3.	3. The diagonals of a parallelogram bisect each other.
4. H is the midpoint of $\overline{RS}$.	4. All right angles are congruent.
5.	5. Definition of midpoint
6. $\overline{OH} \cong \overline{OH}$	6.
7.	7. SSS Congruence Postulate
8. ∠RHO ≅ ∠SHO	8.
9. ∠RHO and ∠SHO are right angles.	9.
10.	10. Perpendicular lines meet to form right angles.

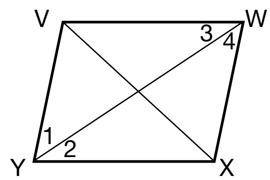
Theorem 4. *Each diagonal of a rhombus bisects opposite angles.*

Show Me!

Given: Rhombus VWXY

Prove: $\angle 1 \cong \angle 2$

$\angle 3 \cong \angle 4$



Proof:

Statements	Reasons
1.	1. Given
2. $\overline{YV} \cong \overline{VW}$; $\overline{WX} \cong \overline{XY}$	2.
3.	3. Reflexive Property
4. $\Delta YVW \cong \Delta WXY$	4.
5. $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$	5.

Note: The proof that VX bisects the other pair of opposite angles is left as an exercise.

You've just done proving the theorems on rectangles and rhombuses. Do you want to know the most special among the kinds of parallelogram and why? Try Activity 10 that follows to help you discover something special

➤ Activity 10: Especially for You

Do the procedures below and answer the questions that follow.

Materials: bond paper, pencil, ruler, protractor, and compass

Procedure:

1. Draw square GOLD. (Note: Clarify how will students draw a square based on its definition: parallelogram with 4 congruent sides and 4 right angles.)
2. Draw diagonals $\overline{GL}$ and $\overline{OD}$ that meet at C.
3. Use a ruler to measure the segments indicated in the table.
4. Use a protractor to measure the angles indicated in the table.

What to measure	$\angle$GDL	$\overline{GL}$ and $\overline{OD}$	$\angle$GCO and $\angle$OCL	$\angle$GDO and $\angle$ODL	$\angle$GOD and $\angle$LOD
Measurement					

Questions:

1. What is the measure of ∠GDL?
   a. If ∠GDL is a right angle, can you consider square a rectangle?
   b. If yes, what theorem on rectangle justifies that a square is a rectangle?
2. What can you say about the lengths of $\overline{GL}$ and $\overline{DO}$?
   a. If $\overline{GL}$ and $\overline{DO}$ have the same measures, can you consider a square a rectangle?
   b. If yes, what theorem on rectangles justifies that a square is a rectangle?
3. What can you say about the measures of ∠GCO and ∠OCL?
   a. If $\overline{GL}$ and $\overline{DO}$ meet to form right angles, can you consider a square a rhombus?
   b. If yes, what theorem on rhombuses justifies that a square is a rhombus?
4. What can you say about the measures of ∠GDO and ∠ODL as a pair and ∠GOD and ∠LOD as another pair?
   a. If $\overline{GL}$ divides opposite angles equally, can you consider a square a rhombus?
   b. If yes, what theorem on rhombuses justifies that a square is a rhombus?

Based on your findings, what is the most special among the kinds of parallelogram? Why? Yes, you're right! The *Square* is the most special parallelogram because all the properties of parallelograms and the theorems on rectangles and rhombuses are true to all squares.

QUIZ 2

A. Answer the following statements with always true, sometimes true, or never true.
1. A square is a rectangle.
2. A rhombus is a square.
3. A parallelogram is a square.
4. A rectangle is a rhombus.
5. A parallelogram is a square.
6. A parallelogram is a rectangle.
7. A quadrilateral is a parallelogram.
8. A square is a rectangle and a rhombus.
9. An equilateral quadrilateral is a rhombus.
10. An equiangular quadrilateral is a rectangle.

B. Name all the parallelograms that possess the given.
1. All sides are congruent.
2. Diagonals bisect each other.
3. Consecutive angles are congruent.
4. Opposite angles are supplementary.
5. The diagonals are perpendicular and congruent.

C. Indicate with a check (✓) mark in the table below the property that corresponds to the given quadrilateral.

Property	Quadrilaterals			
	Parallelogram	Rectangle	Rhombus	Square
1. All sides are congruent.				
2. Opposite sides are parallel.				
3. Opposite sides are congruent.				
4. Opposite angles are congruent.				
5. Opposite angles are supplementary.				
6. Diagonals are congruent.				
7. Diagonals bisect each other.				
8. Diagonals bisect opposite angles.				
9. Diagonals are perpendicular to each other.				
10. A diagonal divides a quadrilateral into two congruent Δs.				

After applying the different theorems on rectangle, rhombus and square, you are now ready to prove the Midline Theorem and the theorems on trapezoids and kites. But before that, revisit *Check Your Guess 3* and see if your guesses were right or wrong. How many did you guess correctly?

Can you still remember the different kinds of triangles? Is it possible for a triangle to be cut to form a parallelogram and vice versa? Do you want to know how it is done? What are the different theorems on trapezoids and kites? Let's start by doing *Check Your Guess 4* that follows.

✓ *Check Your Guess 4*

In the table that follows, write T in the second column if your guess on the statement is true; otherwise, write F. You are to revisit the same table later and respond to your guesses by writing R if you were right or W if wrong under the third column.

Statement	My guess is... (T or F)	I was… (R or W)
1. The segment that joins the midpoints of two sides of a triangle is parallel to the third side and half as long.		
2. The median of a trapezoid is parallel to the bases and its length is equal to half the sum of the lengths of the bases.		

3. The base angles of an isosceles trapezoid are congruent.		
4. The legs of an isosceles trapezoid are parallel and congruent.		
5. The diagonals of a kite are perpendicular bisectors of each other.		

The Midline Theorem

➤ Activity 11: It's Paperellelogram!

Form a group of four members and require each member to have the materials needed. Follow the given procedure.

Materials: 4 pieces of short bond paper, pencil, ruler, adhesive tape, protractor, and pair of scissors

Procedure:

1. Each member of the group shall draw and cut a different kind of triangle out of a bond paper. (equilateral triangle, right triangle, obtuse triangle, and acute triangle that is not equiangular)
2. Choose a third side of a triangle. Mark each midpoint of the other two sides then connect the midpoints to form a segment.
   - Does the segment drawn look parallel to the third side of the triangle you chose?
3. Measure the segment drawn and the third side you chose.
   - Compare the lengths of the segments drawn and the third side you chose. What did you observe?
4. Cut the triangle along the segment drawn.
   - What two figures are formed after cutting the triangle along the segment drawn?
5. Use an adhesive tape to reconnect the triangle with the other figure in such a way that their common vertex was a midpoint and that congruent segments formed by a midpoint coincide.
   - After reconnecting the cutouts, what new figure is formed? Why?
   - Make a conjecture to justify the new figure formed after doing the above activity. Explain your answer.
   - What can you say about your findings in relation to those of your classmates?
   - Do you think that the findings apply to all kinds of triangles? Why?

Your findings in Activity 11 helped you discover *The Midline Theorem* as follows:

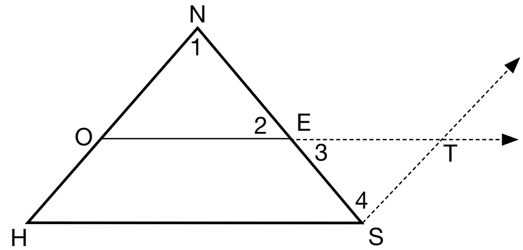
- Theorem 5. The segment that joins the midpoints of two sides of a triangle is parallel to the third side and half as long.

Just like what you did to the theorems on the kinds of parallelogram, *Show Me!* activity to prove the above theorem must be done.

Show Me!

Given: ΔHNS, O is the midpoint of $\overline{HN}$,
E is the midpoint of $\overline{NS}$

Prove: $\overline{OE} \parallel \overline{HS}$, $OE = \frac{1}{2}HS$



Proof:

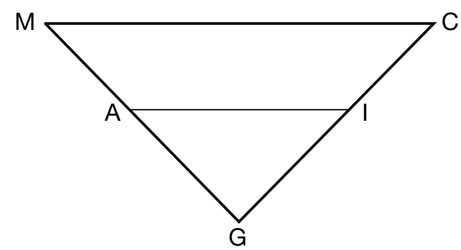
Statements	Reasons
1. ΔHNS, O is the midpoint of $\overline{HN}$, E is the midpoint of $\overline{NS}$	1.
2. In a ray opposite $\overrightarrow{EO}$, there is a point T such that OE = ET	2. In a ray, point at a given distance from the endpoint of the ray.
3. $\overline{EN} \cong \overline{ES}$	3.
4. $\angle 2 \cong \angle 3$	4.
5. $\triangle ONE \cong \triangle TSE$ $\Delta ONE \cong \Delta TSE$	5.
6. $\angle 1 \cong \angle 4$	6.
7. $\overline{HN} \parallel \overline{ST}$	7.
8. $\overline{OH} \cong \overline{ON}$	8.
9. $\overline{ON} \cong \overline{TS}$	9.
10. $\overline{OH} \cong \overline{ST}$	10.
11. Quadrilateral HOTS is a parallelogram.	11.
12. $\overline{OE} \parallel \overline{HS}$	12.
13. OE + ET = OT	13.
14. OE + OE = 0T	14.
15. 2OE = OT	15.
16. $\overline{HS} \cong \overline{OT}$	16.
17. 2OE = HS	17.
18. $OE = \frac{1}{2}HS$ (The segment joining the midpoints of two sides of a triangle is half as long as the third side.)	18.

You've just completed the proof of the Midline Theorem. This theorem can be applied to solve problems. Try the activity that follows.

Solving a Problem Using the Middle Theorem

➤ Activity 12: Go for It!

In ΔMCG, A and I are the midpoints of $\overline{MG}$ and $\overline{GC}$, respectively. Consider each given information and answer the questions that follow.



1. Given: AI = 10.5
   Questions:
   - What is MC?
   - How did you solve for MC?
2. Given: CG = 32
   Questions:
   - What is GI?
   - How did you solve for GI?
3. Given: AG = 7 and CI = 8
   Questions:
   - What is MG + GC?
   - How did you solve for the sum?
4. Given: AI = $3x - 2$ and MC = $9x - 13$
   Questions:
   - What is the value of x?
   - How did you solve for x?
   - What is the sum of AI + MC? Why?

5. Given: $\overline{MG} \cong \overline{CG}$, AG – $2y - 1$, IC = $y + 5$
   Questions:
   - What is the value of y?
   - How did you solve for y?
   - How long are $\overline{MG}$ and $\overline{CG}$? Why?

Another kind of quadrilateral that is equally important as parallelogram is the trapezoid. A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called the *bases* and the non-parallel sides are called the *legs*. The angles formed by a base and a leg are called *base angles*.
You are to prove some theorems on trapezoids. But before doing a series of *Show Me!* activities, do the following activity.

The Midsegment Theorem of Trapezoid

➤ Activity 13: What a Trap!

Do the procedure below and answer the questions that follow.

Materials: bond paper, pencil, ruler, and protractor

Procedure:

1. Draw trapezoid TRAP where $\overline{TRP} \perp \overline{PA}$, TP = 5 cm, TR = 4 cm, and PA = 8 cm.
2. Name the midpoints of $\overline{TP}$ and $\overline{RA}$ as G and O, respectively.
3. Connect G and O to form a segment.

Questions:

- Does $\overline{GO}$ look parallel to the bases of the trapezoid?
- Measure $\overline{GO}$. How long is it?
- What is the sum of the bases of TRAP?
- Compare the sum of the bases and the length of $\overline{GO}$. What did you find?
- Make a conjecture about the sum of the bases and the length of the segment joined by the midpoints of the legs. Explain your answer.

The segment joining the midpoints of the legs of a trapezoid is called *median*.
Activity 13 helped you discover the following theorem about the median of a trapezoid:

- Theorem 6. The median of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

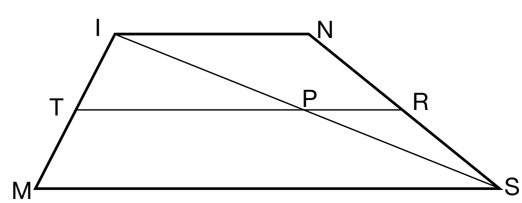
To prove the theorem above, do *Show Me!* activity that follows.

Show Me!

Given: Trapezoid MINS with median $\overline{TR}$

Prove: $\overline{TR} \parallel \overline{IN}$, $\overline{TR} \parallel \overline{MS}$

$$TR = \frac{1}{2}(MS + IN)$$



Proof:

Statements	Reasons
1.	1. Given
2. Draw $\overline{IS}$, with P as its midpoint.	2.
3. $TP = \frac{1}{2}MS$ and $\overline{TP} \parallel \overline{MS}$	3.
4.	4. Theorem 5 (Midline theorem), on △INS

5. $\overline{MS} \parallel \overline{IN}$	5.
6. $\overline{TP} \parallel \overline{IN}$	6.
7. $\overline{TP}$ and $\overline{PR}$ are both parallel to $\overline{TP} \parallel \overline{IN}$. Thus, T, P, and R are collinear.	7.
8. TR = TP + PR	8.
9.	9. Substitution
10. $TR = \frac{1}{2}(MS + IN)$	10.

You've just proven Theorem 6 correctly. Now, what if the legs of the trapezoid become congruent? What must be true about its base angles and its diagonals? Try doing Activity 14 that follows.

Theorems on Isosceles Trapezoid

➤ Activity 14: Watch Out! Another Trap!

Do the procedure below and answer the questions that follow.

Materials: bond paper, pencil, ruler, protractor, and compass

Procedure:

1. On a bond paper, draw rectangle WXIA where WX = 7 cm and WA = 5 cm.
2. On $\overline{WX}$, name a point G 1 cm from W and another point N 1 cm from X.
3. Form $\overline{GA}$ and $\overline{NI}$, to illustrate isosceles trapezoid GAIN. (Note: The teacher of the student has to explain why the figure formed is an isosceles trapepzoid).
4. Use a protractor to measure the four angles of the trapezoid. Record your findings in the table below.
5. Draw the diagonals of GAIN.
6. Use a ruler to measure the diagonals. Record your findings in the table below.

What to measure	∠AGE	∠GAI	∠AIN	∠INW	$\overline{GI}$	$\overline{AN}$
Measurement						

Questions:

1. What two pairs of angles formed are base angles?
2. Compare the measures of the angles in each pair. What did you find?
3. Make a conjecture about the measures of the base angles of an isosceles trapezoid. Explain your answer.

4. Which two pairs of angles are opposite each other?
5. Add the measures of the angles in each pair. What did you find?
6. Make a conjecture about the measures of the opposite angles of an isosceles trapezoid. Explain your answer.
7. Compare the lengths of the diagonals. What did you find?
8. Make a conjecture about the diagonals of an isosceles trapezoid. Explain your answer.

Based on Activity 14, you've discovered three theorems related to isosceles trapezoids as follows:

- Theorem 7. The base angles of an isosceles trapezoid are congruent.
- Theorem 8. Opposite angles of an isosceles trapezoid are supplementary.
- Theorem 9. The diagonals of an isosceles trapezoid are congruent.

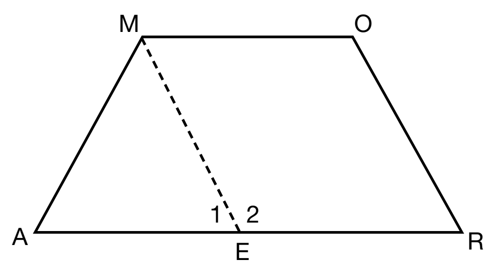
Theorem 7. *The base angles of an isosceles trapezoid are congruent.*

Show Me!

Given: Isosceles Trapezoid AMOR

MO//AR

Prove: ∠A ≅ ∠R, ∠AMO ≅ ∠O



Proof:

Statements	Reasons
1.	1. Given
2. $\overline{AM} \cong \overline{OR}$; $\overline{MO} \parallel \overline{AR}$	2.
3. From M, draw $\overline{ME} \parallel \overline{OR}$ where E lies on $\overline{AR}$.	3.
4.	4. Definition of a parallelogram
5. $\overline{ME} \cong \overline{OR}$	5.
6. $\overline{OR} \cong \overline{ME}$	6.
7.	7. Transitive Property (SN 2 and 6)
8. ΔAME is an isosceles triangle.	8.
9. ∠1 ≅ ∠A	9.
10. ∠1 ≅ ∠R	10.
11. ∠R ≅ ∠A	11.

12. $\angle A \cong \angle R$	12.
13. $\angle A$ and $\angle AMO$ are supplementary angles. $\angle O$ and $\angle R$ are supplementary angles.	13.
14. $\angle AMO \cong \angle O$	14.

Theorem 7 is proven true. You may proceed to the next *Show Me!* activities to prove Theorem 8 and Theorem 9.

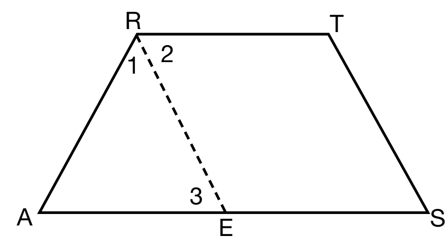
Theorem 8. *Opposite angles of an isosceles trapezoid are supplementary.*

Show Me!

Given: Isosceles Trapezoid ARTS

Prove: $\angle ARS$ and $\angle S$ are supplementary.

$\angle A$ and $\angle T$ are supplementary.



Proof:

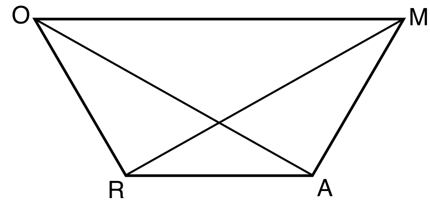
1.	1. Given
2. $\overline{AR} \cong \overline{TS}$; $\overline{RT} \cong \overline{AS}$	2.
3. From R, draw $\overline{RE} \parallel \overline{TS}$ where E lies on $\overline{AS}$.	3.
4.	4. Definition of a parallelogram
5. $\overline{TS} \cong \overline{RE}$	5.
6.	6. Transitive Property
7. ΔARE is an isosceles triangle.	7.
8. $\angle 3 \cong \angle A$	8.
9. $m\angle 1 + m\angle 3 + m\angle A$	9.
10. $\angle 3 \cong \angle 2$	10.
11. $\angle A \cong \angle S$	11.
12. $m\angle 1 + m\angle 2 + m\angle S$	12.
13. $\angle 1 + \angle 2 = \angle ART$	13.
14. $m\angle ART + m\angle S$	14.
15. $m\angle S + m\angle T$	15.
16. $m\angle A + m\angle T$	16.
17. $\angle ART$ and $\angle S$ are supplementary; $\angle A$ and $\angle T$ are supplementary	17.

Theorem 9. *The diagonals of an isosceles trapezoid are congruent.*

Show Me!

Given: Isosceles Trapezoid ROMA

Prove: $\overline{RM} \cong \overline{AO}$



Proof:

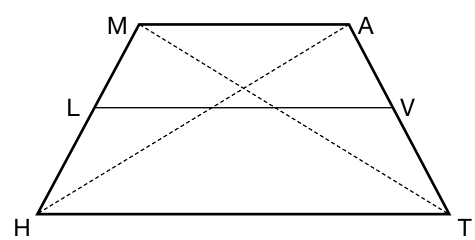
Statements	Reasons
1.	1. Given
2. $\overline{OR} \cong \overline{MA}$	2.
3. $\angle ROM \cong \angle AMO$	3.
4. $\overline{OM} \cong \overline{MO}$	4.
5.	5. SAS Congruence Postulate
6. $\overline{RM} \cong \overline{AO}$	6.

Solving Problems Involving Theorems on Trapezoids

➤ Activity 15: You Can Do It!

Consider the figure on the right and answer the questions that follow.

Given: Quadrilateral MATH is an isosceles trapezoid with bases $\overline{MA}$ and $\overline{HT}$, $\overline{LV}$ is a median.



1. Given: MA = $3y - 2$; HT = $2y + 4$; LV = 8.5 cm

   Questions:

   - What is the value of y?
   - How did you solve for y?
   - What are MA and HT?

2. Given: $\angle$HMA = 115 m

   Questions:

   - What is m $\angle$TAM?
   - What theorem justifies your answer?

3. Given: m ∠MHT = $3x + 10$; m ∠MAT = $2x – 5$m

   Questions:
   - What is the value of x?
   - How did you solve for x?
   - What are the measures of the two angles?
   - What theorem justifies your answer?

4. Given: AH = $4y – 3$; MT = $2y + 5$

   Questions:
   - What is the value of y?
   - How did you solve for y?
   - How long is each diagonal?
   - What theorem justifies your answer above?

> You've just applied the different theorems concerning trapezoids. Now, you will prove another set of theorems, this time concerning kites. Have you ever experienced making a kite? Have you tried joining a kite festival in your community? A *kite* is defined as quadrilateral with two pairs of adjacent and congruent sides. Note that a rhombus (where all adjacent sides are equal) is a special kind of kite.

Theorems on Kite

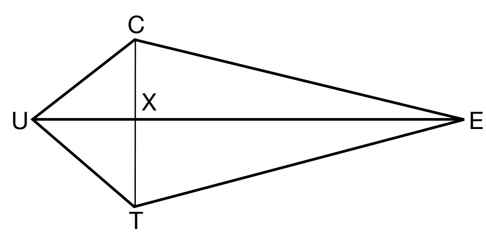
➤ Activity 16: Cute Kite

Do the procedure below and answer the questions that follow.

Materials: bond paper, pencil, ruler, protractor, compass, and straightedge

Procedure:

1. Draw kite CUTE where $\overline{UC} \cong \overline{UT}$ and $\overline{CE} \cong \overline{TE}$ like what is shown at the right. Consider diagonals $\overline{CT}$ and $\overline{UE}$ that meet at X.



2. Use a protractor to measure each of the angles with vertex at X. Record your findings in the table below.
3. Use a ruler to measure the indicated segments and record your findings in the table below.

What to measure	∠CXU	∠UXT	∠EXT	∠CXE	∠CXE	$\overline{CX}$	$\overline{XT}$
Measurement							

Questions:

- What do you observe about the measures of the angles above?
- How are the diagonals related to each other?
- Make a conjecture about the diagonals of a kite based on the angles formed. Explain your answer.
- Compare the lengths of the segments given above. What do you see?
- What does $\overline{UE}$ do to $\overline{CT}$ at X? Why?
- Make a conjecture about the diagonals of a kite based on the pair of congruent segments formed. Explain your answer.

> There are two theorems related to kites as follows:
> Theorem 10. In a kite, the perpendicular bisector of at least one diagonal is the other diagonal.
> Theorem 11. The area of a kite is half the product of the lengths of its diagonals.

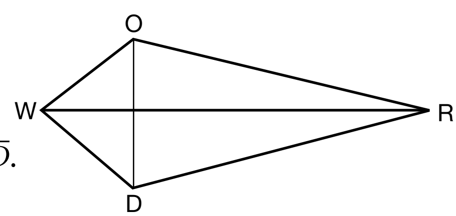
Theorem 10. *In a kite, the perpendicular bisector of at least one diagonal is the other diagonal.*

Show Me!

Given: Kite WORD with diagonals $\overline{WR}$ and $\overline{OD}$

Prove: $\overline{WR}$ is the perpendicular bisector of $\overline{OD}$.

Proof:



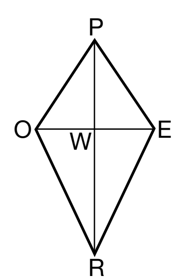
Statements	Reasons
1.	1. Given
2. $\overline{WO} \cong \overline{WD}$; $\overline{OR} \cong \overline{DR}$	2.
3. WO = WD ; OR = DR	3.
4.	4. If a line contains two points each of which is equidistant from the endpoints of a segment, then the line is the perpendicular bisector of the segment.

Theorem 11. *The area of a kite is half the product of the lengths of its diagonals.*

Show Me!

Given: Kite ROPE

Prove: Area of kite ROPE = $\frac{1}{2}(OE)(PR)$



Proof:

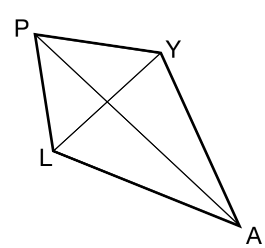
Statements	Reasons
1.	1. Given
2.	2. The diagonals of a kite are perpendicular to each other.
3. Area of kite ROPE = Area of ΔOPE + Area of ΔORE	3. Area Addition Postulate
4. Area of ΔOPE $= \frac{1}{2}$(OE) (PW) Area of ΔORE $= \frac{1}{2}$(OE) (WR)	4. Area Formula for Triangles
5. Area of kite ROPE $= \frac{1}{2}(OE)(PW) + \frac{1}{2}(OE)(WR)$	5.
6. Area of kite ROPE $= \frac{1}{2}(OE)(PW + WR)$	6.
7. PW + WR = PR	7.
8. Area of kite ROPE $= \frac{1}{2}(OE)(PR)$	8.

Solving Problems Involving Kites

➤ Activity 17: Play a Kite

Consider the figure that follows and answer the given questions.

Given: Quadrilateral PLAY is a kite.



1. Given: PA = 12 cm; LY = 6 cm

   Questions:
   - What is the area of kite PLAY?
   - How did you solve for its area?
   - What theorem justifies your answer?

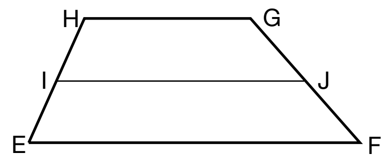
2. Given: Area of kite PLAY = 135 cm^2; LY = 9 cm

   Questions:
   - How long is PA?
   - How did you solve for PA?
   - What theorem justifies your answer above?

It's amazing that the area of a kite has been derived from the formula in finding for the area of a triangle.

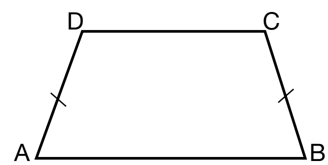
QUIZ 3

A. Refer to trapezoid EFGH with median $\overline{IJ}$



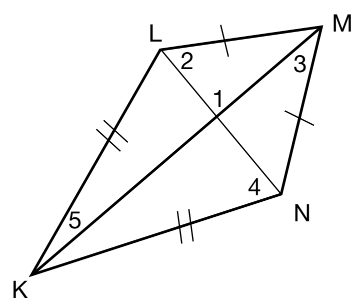
1. If IJ = x, HG = 8 and EF = 12, what is the value of x?
2. If IJ = $y + 3$, HG = 14 and EF = 18, what is the value of y? What is IJ?
3. If HG = x, IJ = 16 and EF = 22, what is value of x?
4. If HG = $y - 2$, IJ = 20 and EF = 31, what is the value of y? What is HG?
5. If HI = 10 and IE = $x - 4$, what is the value of x? What is IE?

B. Given isosceles trapezoid ABCD



1. Name the legs.
2. Name the bases.
3. Name the base angles.
4. If $m\angle A = 70$, what is $m\angle B$?
5. If $m\angle D = 105$, what is $m\angle C$?
6. If $m\angle B = 2x - 6$ and $m\angle A = 82$, what is x?
7. If $m\angle C = 2(y + 4)$ and $m\angle D = 116$, what is y?
8. If AC = 56 cm, what is DB?
9. If AC = $2x + 10$ and DB = $4x - 6$, what is AC?
10. If DB = $3y + 7$ and AC = $6y - 8$, what is DB?

C. Consider kite KLMN on the right.



1. Name the pairs of congruent and adjacent sides.
2. If LM = 6, what is MN?
3. If KN = 10.5, what is KL?
4. If LN = 7 cm and KM = 13 cm, what is the area?
5. If the area is 96 cm2 and LN = 8 cm, what is KM?
6. If m 2 = 63, what is m 3?
7. If $m\angle 3 = 31$, what is $m\angle LMN$?
8. If $m\angle 5 = 22$, what is $m\angle 4$?
9. If $m\angle LKN = 39$, what is $m\angle MKN$?
10. If $m\angle 4 = 70$, what is $m\angle KLN$?

After applying the Midline Theorem and the different theorems on trapezoids and kites, you are now ready to solve problems involving parallelograms, trapezoids, and kites. But before that, revisit *Check Your Guess 4* and see if your guesses were right or wrong. How many did you guess correctly?

What to REFLECT and UNDERSTAND

It's high time for you to further understand concepts you've learned on parallelograms, trapezoids, and kites. Remember the theorems you've proven true for these will be very useful as you go on with the different activities. Your goal in this section is to apply the properties and theorems of the different quadrilaterals in doing the activities that follow. Let's start by doing Activity 18.

➤ Activity 18: You Complete Me!

Write the correct word to complete the crossword puzzle below.

DOWN

1 – quadrilateral ABCD where $\overline{AB} \parallel \overline{CD}$; $\overline{AD} \parallel \overline{BC}$

2 – parallelogram FILM where $\overline{FI} \cong \overline{IL} \cong \overline{LM} \cong \overline{MF}$

3 – a polygon with two diagonals

5 – a condition where two coplanar lines never meet

8 – quadrilateral PARK where $\overline{PR} \perp \overline{AK}$; $\overline{PR} \neq \overline{AK}$

ACROSS

2 – quadrilateral HEAT where $\angle H \cong \angle E \cong \angle A \cong \angle T$

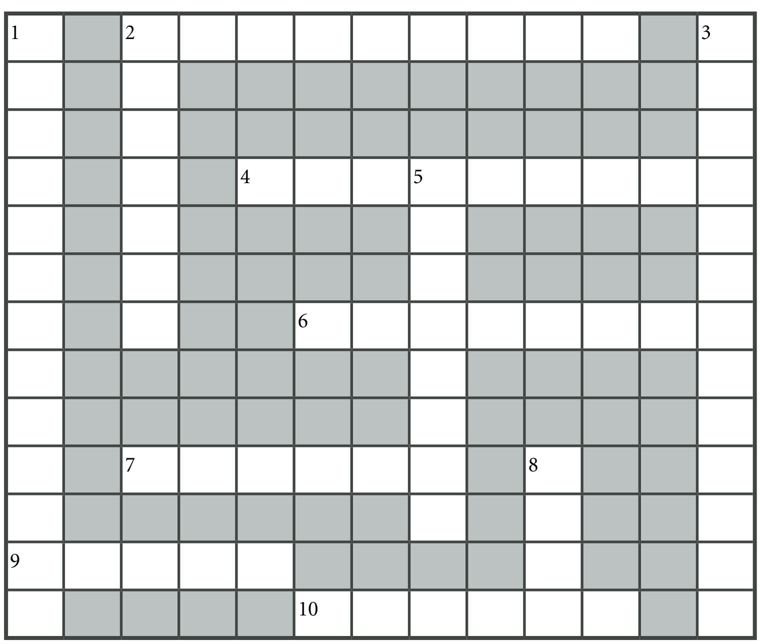
4 – quadrilateral KING where $\overline{KI} \parallel \overline{NG}$ and $\overline{KG}$ is not parallel to $\overline{IN}$

6 – $\overline{RO}$ in quadrilateral TOUR

7 – parallelogram ONLY were $\angle O \cong \angle N \cong \angle L \cong \angle Y$ and $\overline{ON} \cong \overline{NL} \cong \overline{LY} \cong \overline{YO}$

9 – formed by two consecutive sides of a polygon

10 – U in quadrilateral MUSE



➤ Activity 19: It's Showtime!

Graph and label each quadrilateral with the given vertices on a graph paper. Complete the information needed in the table below and answer the questions that follow.

Quadrilateral	Specific Kind
ABCD	
EFGH	
IJKL	
MNOP	
QRST	

1. A(3, 5), B(7, 6), C(6, 2), D(2, 1)
2. E(2, 1), F(5, 4), G(7, 2), H(2, –3)
3. I(–6, –4), J(–6, 1), K(–1, 1), L(–1, –4)
4. M(–1, 1), N(0, 2), O(1, 1), P(0, –2)
5. Q(–2, –3), R(4, 0), S(3, 2), T(–3, –1)

Questions:

1. Which quadrilateral is a rectangle? Why? Verify the following theorems by using the idea of slope. (Hint: Parallel lines have equal slopes while perpendicular lines have slopes whose product is –1.)
   - both pairs of opposite sides are parallel
   - four pairs of consecutive sides are perpendicular
   - diagonals are not necessarily perpendicular to each other
2. Which quadrilateral is a trapezoid? Why? Verify the following theorems by using the idea of slope.
   - one pair of opposite sides are parallel
   - one pair of opposite sides are not parallel
3. Which quadrilateral is a kite? Why? Verify the following theorems by using the idea of slope.
   - both pairs of opposite sides are not parallel
   - diagonals are perpendicular
4. Which quadrilateral is a rhombus? Why? Verify the following theorems by using the idea of slope.
   - both pairs of opposite sides are parallel
   - four pairs of consecutive sides are not necessarily perpendicular
   - diagonals are perpendicular

5. Which quadrilateral is a square? Why? Verify the following theorems by using the idea of slope.
   - both pairs of opposite sides are parallel
   - four pairs of consecutive sides are perpendicular
   - diagonals are perpendicular

Solving Problems Involving Parallelograms, Trapezoids, and Kites

➤ Activity 20: Show More What You've Got!

Solve each problem completely and accurately on a clean sheet of paper. Show your solution and write the theorems or properties you applied to justify each step in the solution process. You may illustrate each given, to serve as your guide. Be sure to box your final answer.

1. Given: Quadrilateral WISH is a parallelogram.
   a. If m $\angle W = x + 15$ and m $\angle S = 2x + 5$, what is m $\angle W$?
   b. If WI = $3y + 3$ and HS = $y + 13$, how long is $\overline{HS}$?
   c. ▭WISH is a rectangle and its perimeter is 56 cm. One side is 5 cm less than twice the other side. What are its dimensions and how large is its area?
   d. What is the perimeter and the area of the largest square that can be formed from rectangle WISH in 1.c.?

2. Given: Quadrilateral POST is an isosceles trapezoid with $\overline{OS} \parallel \overline{PT}$. $\overline{ER}$ is its median.
   a. If OS = $3x - 2$, PT = $2x + 10$ and ER = 14, how long is each base?
   b. If m $\angle P = 2x + 5$ and m $\angle O = 3x - 10$, what is m $\angle T$?
   c. One base is twice the other and ER is 6 cm long. If its perimeter is 27 cm, how long is each leg?
   d. ER is 8.5 in long and one leg measures 9 in. What is its perimeter if one of the bases is 3 in more than the other?

3. Given: Quadrilateral LIKE is a kite with $\overline{LI} \cong \overline{IK}$ and $\overline{LE} \cong \overline{KE}$.
   a. LE is twice LI. If its perimeter is 21 cm, how long is $\overline{LE}$?
   b. What is its area if one of the diagonals is 4 more than the other and IE + LK = 16 in?
   c. IE = $(x - 1)$ ft and LK = $(x + 2)$ ft. If its area is 44 ft^2, how long are $\overline{IE}$ and $\overline{LK}$?

The activities you did above clearly reflect your deeper understanding of the lessons taught to you in this module. Now, you are ready to put your knowledge and skills to practice and be able to answer the question you've instilled in your mind from the very beginning of this module—*"How useful are quadrilaterals in dealing with real-life situations?"*

What to TRANSFER

Your goal in this section is to apply what you have learned to real-life situations. This shall be one of your group outputs for the third quarter. A practical task shall be given to your group where each of you will demonstrate your understanding with accuracy, and further supported through refined mathematical justification along with your projects' stability and creativity. Your work shall be graded in accordance with a rubric prepared for this task.

➤ Activity 21: Fantastic Quadrilatable!

Goal: To design and create a study table having parts showing the different quadrilaterals (out of recyclable materials if possible)

Role: Design engineers

Audience: Mathematics club adviser and all Mathematics teachers

Situation: The Mathematics Club of your school initiated a project entitled "Operation Quadrilatable" for the improvement of your Mathematics Park/Center. Your group is tasked to design and create a study table having parts showing the different quadrilaterals (out of recyclable materials if possible) using Euclidean tools (a compass and a straightedge) and present your output to the Mathematics Club adviser and all Mathematics teachers for evaluation. By having an additional study table in the park, students shall have more opportunities to study their lessons either individually or in groups. In this way, they will continue to learn loving and to love learning Mathematics in particular and all subjects in general.

Product: "Quadrilatable" as study table

Standards: Accuracy, creativity, stability, and mathematical justification

Rubrics for the Performance Task

Criteria	Outstanding (4)	Satisfactory (3)	Developing (2)	Beginning (1)	Rating
Accuracy	The computations are accurate and show wise use of the key concepts in the properties and theorems of all quadrilaterals.	The computations are accurate and show the use of the key concepts in the properties and theorems of all quadrilaterals.	The computations are erroneous and show some use of the key concepts in the properties and theorems of all quadrilaterals.	The computations are erroneous and do not show the use of the key concepts in the properties and theorems of all quadrilaterals.	

Mathematical Justification	The explanation and reasoning are very clear, precise, and coherent. It included facts and principles related to quadrilaterals.	The explanation and reasoning are clear, precise, and coherent. It included facts and principles related to quadrilaterals.	The explanation and reasoning are vague but it included facts and principles related to quadrilaterals.	The explanation and reasoning are vague and it didn't include facts and principles related to quadrilaterals.	
Creativity	The overall impact of the output is very impressive and the use of technology is very evident.	The overall impact of the output is impressive and the use of technology is evident.	The overall impact of the output is fair and the use of technology is evident.	The overall impact of the output is poor and the use of technology is not evident.	
Stability	The output is well-constructed, can stand on itself, and functional.	The output is constructed, can stand on itself, and functional.	The output is constructed, can stand on itself but not functional.	The output is constructed, can't stand on itself and not functional.	

Questions:

1. How do you feel creating your own design of "quadrilatable"?
2. What insights can you share from the experience?
3. Did you apply the concepts on the properties and theorems of quadrilaterals to the surface of the table you've created? How?
4. Can you think of other projects wherein you can apply the properties and theorems of the different quadrilaterals? Cite an example and explain.
5. How useful are the quadrilaterals in dealing with real-life situations? Justify your answer.

Summary/Synthesis/Generalization

This module was about parallelograms, trapezoids, and kites. In this module, you were able to identify quadrilaterals that are parallelograms; determine the conditions that make a quadrilateral a parallelogram; use properties to find measures of angles, sides, and other quantities involving parallelograms; prove theorems on the different kinds of parallelogram (rectangle, rhombus, square); prove the Midline Theorem; and prove theorems on trapezoids and kites. More importantly, you were given the chance to formulate and solve real-life problems, and demonstrate your understanding of the lesson by doing some practical tasks.

You have learned the following:

Conditions Which Guarantee that a Quadrilateral a Parallelogram

1. A quadrilateral is a parallelogram if both pairs of opposite sides are congruent.
2. A quadrilateral is a parallelogram if both pairs of opposite angles are congruent.
3. A quadrilateral is a parallelogram if pairs of consecutive angles are supplementary.
4. A quadrilateral is a parallelogram if the diagonals bisect each other.
5. A quadrilateral is a parallelogram if each diagonal divides a parallelogram into two congruent triangles.
6. A quadrilateral is a parallelogram if one pair of opposite sides are congruent and parallel.

Properties of a Parallelogram

1. In a parallelogram, any two opposite sides are congruent.
2. In a parallelogram, any two opposite angles are congruent.
3. In a parallelogram, any two consecutive angles are supplementary.
4. The diagonals of a parallelogram bisect each other.
5. A diagonal of a parallelogram forms two congruent triangles.

List of Theorems in This Module

Theorems on rectangle:

Theorem 1. If a parallelogram has one right angle, then it has four right angles and the parallelogram is a rectangle.

Theorem 2. The diagonals of a rectangle are congruent.

Theorems on rhombus:

Theorem 3. The diagonals of a rhombus are perpendicular.

Theorem 4. Each diagonal of a rhombus bisects opposite angles.

Theorem 5. *The Midline Theorem.* The segment that joins the midpoints of two sides of a triangle is parallel to the third side and half as long.

Theorem on trapezoid:

Theorem 6. *The Midsegment Theorem.* The median of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

Theorems on isosceles trapezoid:

Theorem 7. The base angles of an isosceles trapezoid are congruent.

Theorem 8. Opposite angles of an isosceles trapezoid are supplementary.

Theorem 9. The diagonals of an isosceles trapezoid are congruent.

Theorems on kite:

Theorem 10. In a kite, the perpendicular bisector of at least one diagonal is the other diagonal.

Theorem 11. The area of a kite is half the product of the lengths of its diagonals.

Glossary of Terms

adjacent angles – two angles sharing a common side and vertex but no interior points in common

base angles – angles formed by a base and the legs

complementary angles – two angles whose sum of the measures is 90°

diagonal – a line segment joining two nonconsecutive vertices of a polygon

isosceles trapezoid – a trapezoid with congruent legs

kite – a quadrilateral with two pairs of congruent and adjacent sides

median of a trapezoid – the segment joining the midpoints of the legs

parallelogram – a quadrilateral with two pairs of opposite sides that are parallel

quadrilateral – a closed plane figure consisting of four line segments or sides

rectangle – a parallelogram with four right angles

rhombus – a parallelogram with all four sides congruent

right angle – an angle with a measure of 90°

square – a rectangle with all four sides congruent

supplementary angles – two angles whose sum of the measures is 180°

theorem – a statement that needs to be proven before being accepted

trapezoid – a quadrilateral with exactly one pair of opposite sides parallel

vertical angles – two nonadjacent angles formed by two intersecting lines

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