I. INTRODUCTION AND FOCUS QUESTIONS

You have learned special products and factoring polynomials in Module 1. Your knowledge on these will help you better understand the lessons in this module.

Have you ever asked yourself how many people are needed to complete a job? What are the bases for their wages? And how long can they finish the job? These questions may be answered using rational algebraic expressions which you will learn in this module.

After you finished the module, you should be able to answer the following questions:

a. What is a rational algebraic expression?

b. How will you simplify rational algebraic expressions?

c. How will you perform operations on rational algebraic expressions?

d. How will you model rate–related problems?

II. LESSONS AND COVERAGE

In this module, you will examine the abovementioned questions when you take the following lessons:

Lesson 1 – Rational Algebraic Expressions
Lesson 2 – Operations on Rational Algebraic Expressions
In these lessons, you will learn to:

<table>
<thead>
<tr>
<th>Lesson 1</th>
<th>Lesson 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>• describe and illustrate rational algebraic expressions;</td>
<td>• multiply, divide, add and subtract rational algebraic expressions;</td>
</tr>
<tr>
<td>• interpret zero and negative exponents;</td>
<td>• simplify complex fractions; and</td>
</tr>
<tr>
<td>• evaluate algebraic expressions involving integral exponents; and</td>
<td>• solve problems involving rational algebraic expressions.</td>
</tr>
<tr>
<td>• simplify rational algebraic expressions.</td>
<td></td>
</tr>
</tbody>
</table>

Here is a simple map of the lessons that will be covered in this module.
III. PRE - ASSESSMENT

Find out how much you already know about this module. Write the letter that you think is the best answer to each question in a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through in this module.

1. Which of the following expressions is a rational algebraic expression?
   a. \( \dfrac{x}{\sqrt{3y}} \)  
   b. \( \dfrac{3c^{-3}}{\sqrt{(a + 1)^0}} \)  
   c. \( 4y^2 + z^{-3} \)  
   d. \( \dfrac{a - b}{b + a} \)

2. What is the value of a non – zero polynomial raised to 0?
   a. constant  
   b. zero  
   c. undefined  
   d. cannot be determined

3. What will be the result when \( a \) and \( b \) are replaced by 2 and -1, respectively, in the expression \( (-5a^2b)(-2a^{-3}b^2) \)?
   a. \( \dfrac{27}{16} \)  
   b. \( -\dfrac{5}{16} \)  
   c. \( \dfrac{3}{7} \)  
   d. \( -\dfrac{2}{7} \)

4. What rational algebraic expression is the same as \( \dfrac{x^2 - 1}{x - 1} \)?
   a. \( x + 1 \)  
   b. \( x - 1 \)  
   c. 1  
   d. -1

5. When a rational algebraic expression is subtracted from \( \dfrac{3}{x - 5} \), the result is \( \dfrac{-x - 10}{x^2 - 5x} \). What is the other rational algebraic expression?
   a. \( \dfrac{x}{4} \)  
   b. \( \dfrac{x}{x - 5} \)  
   c. \( \dfrac{2}{x} \)  
   d. \( \dfrac{-2}{x - 5} \)
6. Find the product of \( \frac{a^2 - 9}{a^2 + a - 20} \) and \( \frac{a^2 - 8a + 16}{3a - 9} \).

   a. \( \frac{a}{a - 1} \)  
   b. \( \frac{a^2 - 1}{1 - a} \)  
   c. \( \frac{a^2 - 7a + 12}{3a + 15} \)  
   d. \( \frac{a^2 - 1}{a^2 - a + 1} \)

7. What is the simplest form of \( \frac{2}{b-3} \)?

   a. \( \frac{2}{5-b} \)  
   b. \( \frac{b+5}{4} \)  
   c. \( \frac{1}{b-1} \)  
   d. \( \frac{1-b}{3} \)

8. Perform the indicated operation: \( \frac{x - 2}{3} - \frac{x + 2}{2} \).

   a. \( x + 5 \)  
   b. \( x + 1 \)  
   c. \( x - 6 \)  
   d. \( -x - 10 \)

9. The volume of a certain gas will increase as the pressure applied to it decreases. This relationship can be modelled using the formula:

   \[ V_2 = \frac{V_1 P_1}{P_2} \]

   where \( V_1 \) is the initial volume of the gas, \( P_1 \) is the initial pressure, \( P_2 \) is the final pressure, and the \( V_2 \) is the final volume of the gas. If the initial volume of the gas is 500 ml and the initial pressure is \( \frac{1}{2} \) atm, what is the final volume of the gas if the final pressure is 5 atm?

   a. 10ml  
   b. 50ml  
   c. 90ml  
   d. 130ml

10. Angelo can complete his school project in \( x \) hours. What part of the job can be completed by Angelo after 3 hours?

    a. \( x + 3 \)  
    b. \( x - 3 \)  
    c. \( \frac{x}{3} \)  
    d. \( \frac{3}{x} \)

11. If Maribel (Angelo’s groupmate in number 10), can do the project in three hours, which expressions below represents the rate of Angelo and Maribel working together?

    a. \( 3 + x \)  
    b. \( x - 3 \)  
    c. \( \frac{1}{3} - \frac{1}{x} \)  
    d. \( \frac{1}{3} + \frac{1}{x} \)
12. Aaron was asked by his teacher to simplify \( \frac{a^2 - 1}{a^2 - a} \) on the board. He wrote his solution on the board this way:

\[
\frac{a^2 - 1}{a^2 - a} = \frac{(a+1)(a-1)}{a(a-1)} = 1
\]

Did he arrive at the correct answer?

a. Yes. The expressions that he crossed out are all common factors.

b. Yes. The LCD must be eliminated to simplify the expression.

c. No. \( a^2 \) must be cancelled out so that the answer is \( \frac{1}{a} \).

d. No. \( a \) is not a common factor of numerator

13. Your friend multiplied \( \frac{x - 1}{2 - x} \) and \( \frac{1 + x}{1 - x} \). His solution is presented below:

\[
\frac{x - 1}{2 - x} \cdot \frac{1 + x}{1 - x} = \frac{(x - 1)(1 + x)}{(2 - x)(1 - x)} = \frac{x + 1}{2 - x}
\]

Is his solution correct?

a. No. There is no common factor to both numerator and denominator.

b. No. The multiplier must be reciprocated first before multiplying the expressions.

c. No. Common variables must be eliminated.

d. No. Dividing an expression by its multiplicative inverse is not equal to one.

14. Laiza added two rational algebraic expressions and her solution is presented below.

\[
\frac{4x + 3}{2} + \frac{3x - 4}{3} = \frac{4x + 3 + 3x - 4}{2 + 3} = \frac{7x + 1}{5}
\]

Is there something wrong in her solution?

a. Yes. Solve first the GCF before adding the rational algebraic expressions.

b. Yes. Cross multiply the numerator of the first expression to the denominator of the second expression.

c. Yes. She may express first the expressions as similar fractions.

d. Yes. \( 4x - 4 \) is equal to \( x \)
15. Your father, a tricycle driver, asked you regarding the best motorcycle to buy. What will you do to help your father?

a. Look for the fastest motorcycle.
b. Canvass for the cheapest motorcycle.
c. Find an imitated brand of motorcycle.
d. Search for fuel-efficient type of motorcycle.

16. The manager of So-In Clothesline Corp. asked you, as Human Resource Officer, to hire more tailors to meet the production target of the year. What will you consider in hiring a tailor?

a. Speed and efficiency
b. Speed and accuracy
c. Time conscious and personality
d. Experience and personality

17. You own 3 hectares of land and you want to mow it for farming. What will you do to finish it at a very least time?

a. Rent a small mower.  c. Do kaingin.
b. Hire 3 efficient laborers.  d. Use germicide.

18. Your friend asked you to make a floor plan. As an engineer, what aspects should you consider in doing the plan?

a. Precision and realistic
b. Layout and cost
c. Logical and sufficient
d. Creativity and economical

19. Your SK Chairman planned to construct a basketball court. As a contractor, what will you do to realize the project?

a. Show a budget proposal.
b. Make a budget plan.
c. Present a feasibility study.
d. Give a financial statement.

20. As a contractor in number 19, what is the best action to do in order to complete the project on or before the deadline but still on the budget plan?

a. All laborers must be trained workers.
b. Rent more equipment and machines.
c. Add least charge equipment and machines.
d. There must be equal number of trained and amateur workers.
IV. LEARNING GOALS AND TARGETS

As you finish this module, you will be able to demonstrate understanding of the key concepts of rational algebraic expressions and algebraic expressions with integral exponents. You must be able to present evidences of understanding and mastery of the competencies of this module. Activities must be accomplished before moving to the next topic and you must answer the questions and exercises correctly. Review the topic and ensure that answers are correct before moving to a new topic.

Your target in this module is to formulate real-life problems involving rational algebraic expressions with integral exponents and solve these problems with utmost accuracy using variety of strategies. You must present how you perform, apply and transfer these concepts to the real-life situation.
Lesson 1

Rational Algebraic Expressions

What to Know

Let’s begin the lesson by reviewing some of the previous lessons and gathering your thoughts in the lesson.

Activity 1 MATCH IT TO ME

There are verbal phrases below. Look for the mathematical expression in the figures that corresponds to the verbal phrases.

1. The ratio of number $x$ and four added by two
2. The product of square root of three and the number $y$
3. The square of a added by twice the $a$
4. The sum of $b$ and two less than the square of $b$
5. The product of $p$ and $q$ divided by three
6. One – third of the square of $c$.
7. Ten times a number $y$ increased by six
8. Cube of the number $z$ decreased by nine
9. Cube root of nine less than number $w$
10. Number $h$ raised to four

\[
\begin{align*}
\frac{x}{4} + 2 & \quad \frac{2}{x} - \frac{2}{x^2} \\
\frac{3}{c^2} & \quad \frac{x^2 - 1}{x^2 - 2x + 1} \\
10x + 6 & \quad \frac{10}{y} + 4 \\
pq & \quad \frac{b^2}{(b + 2)} \\
\sqrt[3]{y} & \quad \frac{\sqrt{3}y}{y} \\
\frac{b^2 - (b + 2)}{a^2 + 2a} & \quad \frac{\sqrt{3}y}{y} \\
9 - \frac{1}{w^2} & \quad \frac{2}{z^3} \\
\end{align*}
\]
1. What did you feel while translating verbal phrases to mathematical expressions?
2. What must be considered in translating verbal phases to mathematical phrases?
3. Will you consider these mathematical phases as polynomial? Why yes or why not?
4. How will you describe a polynomial?

The previous activity deals with translating verbal phrases to polynomials. You also encountered some examples of non-polynomials. Such activity in translating verbal phases to polynomials is one of the key concepts in answering word problems.

All polynomials are expressions but not all expressions are polynomials. In this lesson you will encounter some of these expressions that are not polynomials.

**Activity 2: HOW FAST**

Suppose you are to print a 40-page research paper. You observed that printer A in the internet shop finished printing it in two minutes.

a. How long do you think printer A can finish 100 pages?
b. How long will it take printer A to finish printing the \( p \) pages?
c. If printer B can print \( x \) pages per minute, how long will printer B take to print \( p \) pages?

**QUESTIONS**

1. Can you answer the first question? If yes, how will you answer it? If no, what must you do to answer the question?
2. How will you describe the second and third questions?
3. How will you model the above problem?

Before moving to the lesson, you have to fill in the table on the next page regarding your ideas on rational algebraic expressions and algebraic expressions with integral exponents.
Activity 3 KWLH

Write your ideas on the rational algebraic expressions and algebraic expressions with integral exponents. Answer the unshaded portion of the table and submit it to your teacher.

<table>
<thead>
<tr>
<th>What I Know</th>
<th>What I Want to Find Out</th>
<th>What I Learned</th>
<th>How Can I Learn More</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

You were engaged in some of the concepts in the lesson but there are questions in your mind. The next section will answer your queries and clarify your thoughts regarding the lesson.

What to Process

Your goal in this section is to learn and understand the key concepts on rational algebraic expressions and algebraic expressions with integral exponents.

As the concepts on rational algebraic expressions and algebraic expressions with integral exponents become clear to you through the succeeding activities, do not forget to think about how to apply these concepts in real-life problems especially to rate-related problems.

Activity 4 (Refer to Activity 1)

MATCH IT TO ME – REVISITED

1. What are the polynomials in the activity “Match It To Me”? List these polynomials under set P.
2. Describe these polynomials.
3. In the activity, which are not polynomials? List these non-polynomials under set R.
4. How do these non-polynomials differ from the polynomial?
5. Describe these non-polynomials.
Activity 5 COMPARE AND CONTRAST

Use your answers in the activity “Match It To Me – Revisited” to complete the graphic organizer compare and contrast. Write the similarities and differences between polynomials and non-polynomials in the first activity.

POLYNOMIALS

NON - POLYNOMIALS

How Alike?

How Different?

In terms of ...

In the activity “Match It to Me”, the non – polynomials are called **rational algebraic expressions**. Your observations regarding the difference between polynomials and non – polynomials in activities 4 and 5 are the descriptions of rational expression. Now, can you define rational algebraic expressions? Write your own definition about rational algebraic expressions in the chart below.
Activity 6: MY DEFINITION CHART

Write your initial definition on rational algebraic expressions in the appropriate box. Your final definition will be written after some activities.

Try to firm up your own definition regarding the rational algebraic expressions by doing the next activity.

Activity 7: CLASSIFY ME

Classify the different expressions below into rational algebraic expression or not rational algebraic expression. Write the expression in the appropriate column.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Rational Algebraic Expressions</th>
<th>Not Rational Algebraic Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{m + 2}{\sqrt{2}}$</td>
<td>$\frac{k}{3k^2 - 6k}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{y + 2}{y - 2}$</td>
<td>$\frac{1}{a^6}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{a}{y^2 - x^2}$</td>
<td>$\frac{1 - m}{m^3}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{c}{a - 2}$</td>
<td>$\frac{c^4}{\sqrt{5}}$</td>
<td></td>
</tr>
</tbody>
</table>

Questions?

1. How many expressions did you place in the column of rational algebraic expression?
2. How many expressions did you place under the column not rational algebraic expression column?
3. How did you classify a rational algebraic expression from a not rational algebraic expression?
4. Were you able to place each expression in its appropriate column?
5. What difficulty did you encounter in classifying the expressions?
In the first few activities, you might have some confusions regarding rational algebraic expressions. However, this section firmed up your idea regarding rational algebraic expressions. Now, put into words your final definition of rational algebraic expression.

**Activity 8**

**MY DEFINITION CHART**

Write your final definition on rational algebraic expressions in the appropriate box.

<table>
<thead>
<tr>
<th>My Initial Definition</th>
<th>My Final Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Compare your initial definition with your final definition of rational algebraic expressions. Are you clarified with your conclusion by the final definition. How? Give at least three rational algebraic expressions differ from your classmate.

**Remember:**

Rational algebraic expression is a ratio of two polynomials provided that the numerator is not equal to zero. In symbols: \( \frac{P}{Q} \), where \( P \) and \( Q \) are polynomials and \( Q \neq 0 \).

In the activities above, you had encountered the rational algebraic expressions. You might encounter some algebraic expressions with negative or zero exponents. In the next activities, you will define the meaning of algebraic expressions with integral exponents including negative and zero exponents.
Complete the table below and observe the pattern.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>2•2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3•3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4•4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
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<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
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<td>5</td>
</tr>
</tbody>
</table>

**RECALL LAWS OF EXPONENTS**

I – Product of Powers
If the expressions multiplied have the same base, add the exponents.

\[ x^a \cdot x^b = x^{a+b} \]

II – Power of a Power
If the expression raised to a number is raised by another number, multiply the exponents.

\[(x^a)^b = x^{ab} \]

III – Power of a Product
If the multiplied expressions is raised by a number, multiply the exponents then multiply the expressions.

\[(x \cdot y)^a = x^a \cdot y^a \quad (x \cdot y)^a = x^a \cdot y^a \]

IV – Quotient of Power
If the ratio of two expressions is raised to a number, then

Case I. \[ \frac{x^a}{x^b} = x^{a-b} \], where \( a > b \)

Case II. \[ \frac{x^a}{x^b} = x^{b-a} \], where \( a < b \)

**Activity 9**

Let the pattern answer it

**Questions**

1. What do you observe as you answer column B?
2. What do you observe as you answer column C?
3. What happens to its value when the exponent decreases?
4. In the column B, how is the value in the each cell/box related to its upper or lower cell/box?

Use your observations in the activity above to complete the table below.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>2</td>
<td>2^5</td>
<td>32</td>
<td>3^5</td>
<td>243</td>
<td>4^5</td>
</tr>
<tr>
<td>2</td>
<td>2^4</td>
<td>34</td>
<td>3^4</td>
<td>243</td>
<td>4^4</td>
</tr>
<tr>
<td>2</td>
<td>2^3</td>
<td>33</td>
<td>3^3</td>
<td>243</td>
<td>4^3</td>
</tr>
<tr>
<td>2</td>
<td>2^2</td>
<td>3^2</td>
<td>3^2</td>
<td>243</td>
<td>4^2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>243</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>243</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>243</td>
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<td>243</td>
<td>2</td>
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<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>243</td>
<td>3</td>
</tr>
</tbody>
</table>
QUESTIONS

1. What did you observe as you answer column A? column B?
2. What happens to its value when the exponent decreases?
3. In column A, how is the value in each cell/box related to its upper or lower cell/box?
4. What do you observe when the number has zero exponent?
5. When a number raised to zero is it the same as another number raised to zero? Justify your answer.
6. What do you observe about the value of the number raised to a negative integer?
7. What can you say about an expression with negative integral exponent?
8. Do you think it is true to all numbers? Cite some examples?

Exercises
Rewrite each item to expressions with positive exponents.

1. $b^{-4}$
2. $c^{-3}/d^{-8}$
3. $w^3z^{-2}$
4. $n^2m^{-2}$
5. $de^6f$
6. $x+y/(x-y)^0$
7. $(a^8b^8c^{10})^0$
8. $14t^0$
9. $p^0/p^0$
10. $2/(a-b+c)^0$

Activity 10
3-2-1 CHART

Complete the chart below.

3 things you found out
2 interesting things
1 question you still have
Activity 11  WHO IS RIGHT?

Allan and Gina were asked to simplify \( \frac{n^3}{n^4} \). Their solutions are shown below together with their explanation.

<table>
<thead>
<tr>
<th>Allan’s Solution</th>
<th>Gina’s Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{n^3}{n^4} = n^{3-4} = n^{-1} = \frac{1}{n} )</td>
<td>( \frac{n^3}{1} = n^3 \cdot \frac{1}{n^{-4}} = n^7 )</td>
</tr>
</tbody>
</table>

Quotient law was used in my solution. I expressed the exponent of the denominator as a positive integer, then followed the rules in dividing polynomials.

Who do you think is right? Write your explanation in a sheet of paper.

You have learned some concepts of rational algebraic expressions as you performed the previous activities. Now, let us try to use these concepts in different contexts.

Activity 12  SPEEDY MARS

Mars finished the 15-meter dash within three seconds. Answer the questions below.

1. How fast did Mars run?
2. At this rate, how far can Mars run after four seconds? five seconds? six seconds?
3. How many minutes can Mars run for 50 meters? 55 meters? 60 meters?

RECALL

Speed is the rate of moving object as it transfers from one point to another. The speed is the ratio between the distance and time travelled by the object.

QUESTIONS

How did you come up with your answer? Justify your answer.

What you just did was evaluating the speed that Mars run. Substituting the value of the time to your speed, you come up with distance. When you substitute your distance to the formula of the speed, you get the time. This concept of evaluation is the same with evaluating algebraic expressions. Try to evaluate the following algebraic expressions in the next activity.
Find the value of each expression below by evaluation.

<table>
<thead>
<tr>
<th>My Expression</th>
<th>Value of $a$</th>
<th>Value of $b$</th>
<th>My solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^2 + b^3$</td>
<td>2</td>
<td>3</td>
<td>Example: $a^2 + b^3 = 2^2 + 3^3 = 4 + 9 = 13$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
<td>Your solution here:</td>
</tr>
<tr>
<td>$\frac{a^2}{b^3}$</td>
<td>-2</td>
<td>3</td>
<td>Example: $\frac{a^2}{b^3} = \frac{(-2)^2}{3^3} = \frac{3^3}{(-2)^2} = \frac{27}{4}$</td>
</tr>
<tr>
<td>$\frac{a^2}{b^3}$</td>
<td>3</td>
<td>2</td>
<td>Your solution here</td>
</tr>
<tr>
<td>$a^{-1}b^0$</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

**Questions:**

1. What have you observed in the solution of the examples?
2. How did these examples help you to find the value of the expression?
3. How did you find the value of the expression?
Exercises
Evaluate the following algebraic expressions.

1. \(40y^{-1}, \ y = 5\)
2. \(\frac{1}{m^2(m + 4)}, \ m = -8\)
3. \((p^2 - 3)^2, \ p = 1\)
4. \(\frac{(x - 1)^2}{(x + 1)^2}, \ x = 2\)
5. \(y^3 - y^2, \ y = 2\)

Activity 14 BIN - GO

Make a 3 by 3 bingo card. Choose numbers to be placed in your bingo card from the numbers below. Your teacher will give an algebraic expression with integral exponents and the value of its variable. The first student who forms a frame wins the game.

<table>
<thead>
<tr>
<th></th>
<th>(\frac{17}{4})</th>
<th>2</th>
<th>(-\frac{31}{8})</th>
<th>(\frac{1}{15})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{2}{9})</td>
<td>(\frac{3}{4})</td>
<td>(\frac{37}{4})</td>
<td>25</td>
</tr>
<tr>
<td>(\frac{1}{11})</td>
<td>(\frac{1}{3})</td>
<td>(\frac{3}{2})</td>
<td>32</td>
<td>2</td>
</tr>
<tr>
<td>(\frac{1}{5})</td>
<td>5</td>
<td>0</td>
<td>(\frac{23}{4})</td>
<td>(\frac{4}{3})</td>
</tr>
<tr>
<td>(\frac{1}{4})</td>
<td>9</td>
<td>0</td>
<td>(\frac{126}{5})</td>
<td>6</td>
</tr>
</tbody>
</table>

The frame card must be like this:

Activity 15 QUIZ CONSTRUCTOR

Be like a quiz constructor. Write in a one-half crosswise three algebraic expressions with integral exponents in at least two variables and decide what values to be assigned in the variables. Show how to evaluate your algebraic expressions. Your algebraic expressions must be unique from your classmates.
Activity 16

CONNECT TO MY EQUIVALENT

Match column A to its equivalent simplest fraction in column B.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{5}{20})</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>(\frac{8}{12})</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>(\frac{4}{8})</td>
<td>(\frac{3}{4})</td>
</tr>
<tr>
<td>(\frac{5}{15})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>(\frac{6}{8})</td>
<td>(\frac{2}{3})</td>
</tr>
</tbody>
</table>

QUESTIONS

1. How did you find the equivalent fractions in column A?
2. Do you think you can apply the same concept in simplifying a rational algebraic expression?

You might wonder how to answer the last question but the key concept of simplifying rational algebraic expressions is the concept of reducing fractions to its simplest form. Examine and analyze the following examples.

Illustrative example: Simplify the following rational algebraic expressions.

1. \(\frac{4a + 8b}{12}\)

Solution
\[
\begin{align*}
\frac{4a + 8b}{12} &= \frac{4(a + 2b)}{4 \cdot 3} \\
&= \frac{a + 2b}{3}
\end{align*}
\]

What factoring method is used in this step?
2. \[ \frac{15c^3d^4e}{12c^2d^3w} \]

Solution
\[ \frac{15c^3d^4e}{12c^2d^3w} = \frac{3 \cdot 5c^3d^4e}{3 \cdot 4c^2d^3dw} \]
\[ = \frac{5ce}{4dw} \]

What factoring method is used in this step?

3. \[ \frac{x^2 + 3x + 2}{x^2 - 1} \]

Solution
\[ \frac{x^2 + 3x + 2}{x^2 - 1} = \frac{(x + 1)(x + 2)}{(x + 1)(x - 1)} \]
\[ = \frac{(x + 2)}{(x - 1)} \]

What factoring method is used in this step?

Based on the above examples,
1. What is the first step in simplifying rational algebraic expressions?
2. What happens to the common factors of numerator and denominator?

Exercises

Simplify the following rational algebraic expressions.

1. \[ \frac{y^2 + 5x + 4}{y^2 - 3x - 4} \]
2. \[ \frac{-21a^2b^2}{28a^3b^3} \]
3. \[ \frac{x^2 - 9}{x^2 - 7x + 12} \]
4. \[ \frac{m^2 + 6m + 5}{m^2 - m - 2} \]
5. \[ \frac{x^2 - 5x - 14}{x^2 + 4x + 4} \]
**Activity 17 MATCH IT DOWN**

Match the rational algebraic expressions to its equivalent simplified expression from the top. Write it in the appropriate column. If the equivalent is not among the choices, write it in column F.

- a. -1  
- b. 1  
- c. a + 5  
- d. 3a  
- e. \( \frac{a}{3} \)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{a^2 + 6a + 5}{a + 1} )</td>
<td>( \frac{a^3 + 2a^2 + a}{3a^2 + 6a + 3} )</td>
<td>( \frac{3a^2 - 6a}{a - 2} )</td>
<td>( \frac{a - 1}{1 - a} )</td>
<td>( \frac{(3a + 2)(a + 1)}{3a^2 + 5a + 2} )</td>
<td>( \frac{3a^3 - 27a}{(a + 3)(a - 3)} )</td>
</tr>
</tbody>
</table>

**Activity 18 CIRCLE PROCESS**

In each circle write steps in simplifying rational algebraic expression. You can add or delete circles if necessary.

---

_In this section, the discussions were all about introduction on rational algebraic expressions. How much of your initial ideas are found in the discussion? Which ideas are different and need revision? Try to move a little further in this topic through the next activities._
Your goal in this section is to relate the operations of rational expressions to real-life problems, especially rate problems.

Work problems are one of the rate-related problems and usually deal with persons or machines working at different rates or speed. The first step in solving these problems involves determining how much of the work an individual or machine can do in a given unit of time called the rate.

Illustrative example:

A. Nimfa can paint the wall in five hours. What part of the wall is painted in three hours?

Solution:

Since Nimfa can paint in five hours, then in one hour, she can paint $\frac{1}{5}$ of the wall. Her rate of work is $\frac{1}{5}$ of the wall each hour. The rate of work is the part of a task that is completed in 1 unit of time.

Therefore, in three hours, she will be able to paint $3 \cdot \frac{1}{5} = \frac{3}{5}$ of the wall.

You can also solve the problem by using a table. Examine the table below.

<table>
<thead>
<tr>
<th>Rate of work (wall painted per hour)</th>
<th>Time worked</th>
<th>Work done (Wall painted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{5}$</td>
<td>1 hour</td>
<td>$\frac{1}{5}$</td>
</tr>
<tr>
<td>$\frac{1}{5}$</td>
<td>2 hours</td>
<td>$\frac{2}{5}$</td>
</tr>
<tr>
<td>$\frac{1}{5}$</td>
<td>3 hours</td>
<td>$\frac{3}{5}$</td>
</tr>
</tbody>
</table>
You can also illustrate the problem.

<table>
<thead>
<tr>
<th>1st hour</th>
<th>2nd hour</th>
<th>3rd hour</th>
<th>4th hour</th>
<th>5th hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{5}$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{1}{5}$</td>
</tr>
</tbody>
</table>

So after three hours, Nimfa only finished painting $\frac{3}{5}$ of the wall.

B. Pipe A can fill a tank in 40 minutes. Pipe B can fill the tank in $x$ minutes. What part of the tank is filled if either of the pipes is opened in ten minutes?

Solution:

Pipe A fills $\frac{1}{40}$ of the tank in 1 minute. Therefore, the rate is $\frac{1}{40}$ of the tank per minute. So after 10 minutes,

$$10 \times \frac{1}{40} = \frac{1}{4}$$

of the tank is full.

Pipe B fills $\frac{1}{x}$ of the tank in $x$ minutes. Therefore, the rate is $\frac{1}{x}$ of the tank per minute. So after $x$ minutes,

$$10 \times \frac{1}{x} = \frac{10}{x}$$

of the tank is full.

In summary, the basic equation that is used to solve work problem is:

Rate of work $\times$ time worked = work done.

$$r \times t = w$$

Activity 19 HOWS FAST 2

Complete the table on the next page and answer questions that follow.

You printed your 40-page reaction paper. You observed that printer A in the internet shop finished printing in two minutes. How long will it take printer A to print 150 pages? How long will it take printer A to print $p$ pages? If printer B can print $x$ pages per minute, how long will it take to print $p$ pages? The rate of each printer is constant.
<table>
<thead>
<tr>
<th>Printer</th>
<th>Pages</th>
<th>Time</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Printer A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 pages</td>
<td>2 minutes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>45 pages</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150 pages</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p )  pages</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Printer B</td>
<td>( p ) pages</td>
<td>( x ) ppm</td>
<td></td>
</tr>
<tr>
<td>30 pages</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35 pages</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 pages</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Questions**

1. How did you solve the rate of each printer?
2. How did you compute the time of each printer?
3. What will happen if the rate of the printer increases?
4. How do time and number of pages affect the rate of the printer?

The concepts of rational algebraic expressions were used to answer the situation above. The situation above gives you a picture how the concepts of rational algebraic expressions were used in solving rate-related problems.

What new realizations do you have about the topic? What new connections have you made for yourself? What questions do you still have? Fill-in the **Learned, Affirmed, Challenged cards** given below.

**Learned**

What new realizations and learning do you have about the topic?

**Affirmed**

What new connections have you made? Which of your old ideas have been confirmed or affirmed?

**Challenged**

What questions do you still have? Which areas seem difficult for you? Which do you want to explore?
Your goal in this section is to apply your learning in real-life situations. You will be given a practical task which will demonstrate your understanding.

Activity 20

The JOB Printing Press has two photocopying machines. P1 can print box of bookpaper in three hours while P2 can print a box of bookpaper in $3x + 20$ hours.

a. How many boxes of bookpaper are printed by P1 in 10 hours? In 25 hours? in 65 hours?

b. How many boxes of bookpaper can P2 print in 10 hours? in $120x + 160$ hours? in $30x^2 + 40x$ hours?

You will show your output to your teacher. Your work will be graded according to mathematical reasoning and accuracy.

RUBRICS FOR YOUR OUTPUT

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>Outstanding 4</th>
<th>Satisfactory 3</th>
<th>Developing 2</th>
<th>Beginning 1</th>
<th>RATING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical reasoning</td>
<td>Explanation shows thorough reasoning and insightful justifications.</td>
<td>Explanation shows substantial reasoning.</td>
<td>Explanation shows gaps in reasoning.</td>
<td>Explanation shows illogical reasoning.</td>
<td></td>
</tr>
<tr>
<td>Accuracy</td>
<td>All computations are correct and shown in detail.</td>
<td>All computations are correct.</td>
<td>Most of the computations are correct.</td>
<td>Some the computations are correct.</td>
<td>OVERALL RATING</td>
</tr>
</tbody>
</table>
Lesson 2
Operations of Rational Algebraic Expressions

What to Know

In the first lesson, you learned that rational algebraic expression is a ratio of two polynomials where the denominator is not equal to zero. In this lesson, you will be able to perform operations on rational algebraic expressions. Before moving to the new lesson, let’s look back on the concepts that you have learned that are essential to this lesson.

In the previous mathematics lesson, your teacher taught you how to add and subtract fractions. What mathematical concept plays a vital role in adding and subtracting fraction? You may think of LCD or Least Common Denominator. Now, let us take another perspective in adding or subtracting fractions. Ancient Egyptians had special rules in their fraction. If they have five loaves for eight persons, they would not divide it immediately by eight instead, they would use the concept of unit fraction. Unit fraction is a fraction with one as numerator. Egyptian fractions used unit fractions without repetition except $\frac{2}{3}$. To be able to divide five loaves among eight persons, they had to cut the four loaves into two and the last one would be cut into eight parts. In short:

$$\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$$

**Activity 1 EGYPTIAN FRACTION**

Now, be like an Ancient Egyptian. Give the unit fractions in Ancient Egyptian way.

1. $\frac{7}{10}$ using two unit fractions.  
2. $\frac{8}{15}$ using two unit fractions.  
3. $\frac{3}{4}$ using two unit fractions.  
4. $\frac{11}{30}$ using two unit fractions.  
5. $\frac{7}{12}$ using two unit fractions.  
6. $\frac{13}{12}$ using three unit fractions.  
7. $\frac{11}{12}$ using three unit fractions.  
8. $\frac{31}{30}$ using three unit fractions.  
9. $\frac{19}{20}$ using three unit fractions.  
10. $\frac{25}{28}$ using three unit fractions.
1. What did you do in getting the unit fraction?
2. How did you feel while getting the unit fractions?
3. What difficulties did you encounter in giving unit fraction?
4. What would you do in overcoming these difficulties?

**Activity 2**

**ANTICIPATION GUIDE**

There are sets of rational algebraic expressions in the table below. Check agree if the entries in column I is equivalent to the entry in column II and check disagree if the entries in the two columns are not equivalent.

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>Agree</th>
<th>Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x^2 - xy}{x^2 - y^2} \cdot \frac{x + y}{x^2 - xy} )</td>
<td>( x^{-1} - y^{-1} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{6y - 30}{y^2 + 2y + 1} + \frac{3y - 15}{y^2 + y} )</td>
<td>( \frac{2y}{y + 1} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{5}{4x^2} + \frac{7}{6x} )</td>
<td>( \frac{15 + 14x}{12x^2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{a}{b - a} - \frac{b}{a - b} )</td>
<td>( \frac{a + b}{b - a} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{a + b}{b} - \frac{b}{a + b} )</td>
<td>( \frac{a^2}{a + b} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Activity 3**

**PICTURE ANALYSIS**

Take a closer look at this picture. Describe what you see.

http://www.portlandground.com/archives/2004/05/volunteers_buil_1.php
1. What would happen if one of them would not do his job?
2. What will happen when there are more people working together?
3. How does the rate of each workers affect the entire work?
4. How will you model the rate-related problem?

The picture above shows how the operations on rational algebraic expressions can be applied to real-life scenario. You’ll get to learn more rate-related problems and how operations on rational algebraic expression relate to it.

What to Process

Your goal in this section is to learn and understand key concepts in the operations on rational algebraic expressions.

As the concepts of operations on rational algebraic expressions become clear to you through the succeeding activities, do not forget to think about how to apply these concepts in solving real-life problems especially rate-related problems.

Activity 4: Multiplying Rational Algebraic Expressions

Examine and analyze the illustrative examples below. Pause once in a while to answer the check-up questions.

The product of two rational expressions is the product of the numerators divided by the product of the denominators. In symbols,

\[ \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \quad bd \neq 0 \]

Illustrative example 1: Find the product of \( \frac{5t}{8} \) and \( \frac{4}{3t^2} \).

\[
\frac{5t}{8} \cdot \frac{4}{3t^2} = \frac{5t \cdot 2^2}{2^3 \cdot 3t^2} = \frac{(5t)(2^2)}{(2^3)(2)(3t^2)}
\]

Express the numerators and denominators into prime factors as possible.
Illustrative example 2: Multiply \( \frac{4x}{3y} \) and \( \frac{3x^2y^2}{10} \).

\[
\frac{4x}{3y} \cdot \frac{3x^2y^2}{10} = \frac{(2)(x)(3)(x^2)(y)(y)}{(3)(y)(5)} = \frac{(2)(x^3)(y)}{5} = \frac{2x^3y}{5}
\]

Illustrative example 3: What is the product of \( \frac{x - 5}{4x^2 - 9} \) and \( \frac{4x^2 + 12x + 9}{2x^2 - 11x + 5} \)?

\[
\frac{x - 5}{4x^2 - 9} \cdot \frac{4x^2 + 12x + 9}{2x^2 - 11x + 5} = \frac{x - 5}{(2x - 3)(2x + 3)} \cdot \frac{(2x - 1)(x - 5)}{(2x - 1)(x - 5)} = \frac{2x + 3}{2x - 3}(2x - 1) = \frac{2x + 3}{4x^2 - 8x + 4}
\]

Questions

1. What are the steps in multiplying rational algebraic expressions?
2. What do you observe from each step in multiplying rational algebraic expressions?

Exercises

Find the product of the following rational algebraic expressions.

1. \( \frac{10uv^2}{3xy^2} \cdot \frac{6x^2y^2}{5u^2v^2} \)
2. \( \frac{a^2 - b^2}{2ab} \cdot \frac{a^2}{a - b} \)
3. \( \frac{x^2 - 3x}{x^2 + 3x - 10} \cdot \frac{x^2 - 4}{x^2 - x - 6} \)
4. \( \frac{x^2 + 2x + 1}{y^2 - 2y + 1} \cdot \frac{y^2 - 1}{x^2 - 1} \)
5. \( \frac{a^2 - 2ab + b^2}{a^2 - 1} \cdot \frac{a - 1}{a - b} \)
Activity 5  WHAT’S MY AREA?

Find the area of the plane figures below.

a. [Image]

b. [Image]

c. [Image]

QUESTIONS

1. How did you find the area of the figures?
2. What are your steps in finding the area of the figures?

Activity 6  THE CIRCLE ARROW PROCESS

Based on the steps that you made in the previous activity, make a conceptual map on the steps in multiplying rational algebraic expressions. Write the procedure and other important concepts in every step inside the circle. If necessary, add a new circle.

Web – based Booster:

Web – based Booster:

Step 1

Step 2

Step 3

Step 4

Final Step

QUESTIONS

1. Does every step have a mathematical concept involved?
2. What makes that mathematical concept important to every step?
3. Can the mathematical concepts used in every step be interchanged? How?
4. Can you give another method in multiplying rational algebraic expressions?
Activity 7  Dividing Rational Algebraic Expressions

Examine and analyze the illustrative examples below. Pause once in a while to answer the check-up questions.

The quotient of two rational algebraic expressions is the product of the dividend and the reciprocal of the divisor. In symbols,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, \quad bc \neq 0$$

Illustrative example 4: Find the quotient of \(\frac{6ab^2}{4cd}\) and \(\frac{9a^2b^2}{8d^2c}\).

$$\frac{6ab^2}{4cd} \div \frac{9a^2b^2}{8d^2c} = \frac{6ab^2}{4cd} \cdot \frac{8d^2c}{9a^2b^2} = \frac{(2)(3)ab^2}{(2)c} \cdot \frac{(3)}{(3)}a^2b^2 \cdot \frac{(2)}{(3)}\frac{d^2c}{(3)}a^2b^2 = \frac{(2)(3)}{(3)a} = \frac{4c}{3a}$$

Multiply the dividend by the reciprocal of the divisor.

Perform the steps in multiplying rational algebraic expressions.

Illustrative example 5: Divide \(\frac{2x^2 + x - 6}{2x^2 + 7x + 5} \div \frac{x^2 - 2x - 8}{2x^2 - 3x - 20}\)

$$\frac{2x^2 + x - 6}{2x^2 + 7x + 5} \div \frac{x^2 - 2x - 8}{2x^2 - 3x - 20} = \frac{(2x - 3)(x + 2)}{(2x + 5)(x + 1)} \cdot \frac{(x - 4)(2x + 5)}{(x + 2)(x - 4)} = \frac{(2x - 3)(x + 2)(x - 4)(2x + 5)}{(2x + 5)(x + 1)(x - 2)(x - 4)} = \frac{(2x - 3)(x + 1)(x - 2)(x - 4)}{(2x + 5)(x + 1)} = \frac{2x - 3}{x + 1}$$

Why do we need to factor out the numerators and denominators?

What happens to the common factors between numerator and denominator?

REVIEW

Perform the operation of the following fractions.

1. \(\frac{1}{2} \div \frac{3}{4}\)
2. \(\frac{5}{2} \div \frac{9}{4}\)
3. \(\frac{9}{2} \div \frac{3}{4}\)
Exercises

Find the quotient of the following rational algebraic expressions.

1. \[ \frac{81x^2y^3}{36y} + \frac{27y^2z^2}{12xy} \]
2. \[ \frac{2a + 2b}{a^2 + ab} + \frac{4}{a} \]
3. \[ \frac{16x^2 - 9}{6 - 5x - 4x^2} + \frac{16x^2 + 24x + 9}{4x^2 + 11x + 6} \]
4. \[ \frac{x^2 + 2x + 1}{x^2 + 4x + 3} + \frac{x^2 - 1}{x^2 + 2x + 1} \]
5. \[ \frac{x - 1}{x + 1} + \frac{1 - x}{x^2 + 2x + 1} \]

Activity 8 MISSING DIMENSION

Find the missing length of the figures.

1. The area of the rectangle is \( \frac{x^2 - 100}{8} \) while the length is \( \frac{2x^2 + 20}{20} \). Find the height of the rectangle.

2. The base of the triangle is \( \frac{21}{3x - 21} \) and the area is \( \frac{x^2}{35} \). Find the height of the triangle.

Questions

1. How did you find the missing dimension of the figures?
2. Enumerate the steps in solving the problems.
Activity 9

Chain Reaction

Use the Chain Reaction Chart to sequence your steps in dividing rational algebraic expressions. Write the process or mathematical concepts used in each step in the chamber. Add another chamber, if necessary.

Questions:

1. Does every step have a mathematical concept involved?
2. What makes that mathematical concept important to every step?
3. Can mathematical concept in every step be interchanged? How?
4. Can you make another method in dividing rational algebraic expressions? How?

Activity 10

Adding and Subtracting Similar Rational Algebraic Expressions

Examine and analyze the following illustrative examples on the next page. Answer the check-up questions.

In adding or subtracting similar rational expressions, add or subtract the numerators and write the answer in the numerator of the result over the common denominator. In symbols,

\[
\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}, \quad b \neq 0
\]
Illustrative example 6: Add \( \frac{x^2 - 2x - 7}{x^2 - 9} \) and \( \frac{3x + 1}{x^2 - 9} \).

\[
\frac{x^2 - 2x - 7}{x^2 - 9} + \frac{3x + 1}{x^2 - 9} = \frac{x^2 - 2x + 3x - 7 + 1}{x^2 - 9} = \frac{x^2 + x - 6}{x^2 - 9}
\]

Combine like terms in the numerator.

\[
= \frac{(x + 3)(x - 2)}{(x - 3)(x + 3)}
\]

Factor out the numerator and denominator.

\[
= \frac{(x - 2)}{(x + 3)}
\]

\[
= \frac{x - 2}{x + 3}
\]

Illustrative example 7: Subtract \( \frac{-10 - 6x - 5x^2}{3x^2 + x - 2} \) from \( \frac{x^2 + 5x - 20}{3x^2 + x - 2} \).

\[
\frac{x^2 + 5x^2 - 20}{3x^2 + x - 2} - \frac{-10 - 6x - 5x^2}{3x^2 + x - 3} = \frac{x^2 + 5x^2 - 20 - (-10 - 6x - 5x^2)}{3x^2 + x - 2} = \frac{x^2 + 5x - 20 + 10 + 6x + 5x^2}{3x^2 + x - 2} = \frac{x^2 + 5x + 6x - 20 + 10}{3x^2 + x - 2} = \frac{6x^2 + 11x - 10}{3x^2 + x - 2} = \frac{(3x - 2)(2x + 5)}{(3x - 2)(x + 1)} = \frac{2x + 5}{x + 1}
\]

Factor out the numerator and denominator.

Exercises

Perform the indicated operation. Express your answer in simplest form.

1. \( \frac{6}{a - 5} + \frac{4}{a - 5} \)
2. \( \frac{x^2 + 3x - 2}{x^2 - 4} + \frac{x^2 - 2x + 4}{x^2 - 4} \)
3. \( \frac{7}{4x - 1} - \frac{5}{4x - 1} \)
4. \( \frac{x^2 + 3x + 2}{x^2 - 2x + 1} - \frac{3x + 3}{x^2 - 2x + 1} \)
5. \( \frac{x - 2}{x - 1} + \frac{x - 2}{x - 1} \)
Activity 11\n\textbf{ADDING AND SUBTRACTING DISSIMILAR RATIONAL ALGEBRAIC EXPRESSIONS}

Examine and analyze the following illustrative examples below. Answer the check-up questions.

\textbf{Review}
Perform the operation of the following fractions.
1. \( \frac{1}{2} + \frac{4}{3} \)  
2. \( \frac{3}{4} + \frac{2}{3} \)  
3. \( \frac{1}{6} - \frac{2}{9} \)

\textbf{Illustrative Example 8:} Find the sum of \( \frac{5}{18a^4b} \) and \( \frac{2}{27a^3b^2c} \).

\[ \frac{5}{18a^4b} + \frac{2}{27a^3b^2c} = \frac{5}{(3^2)(2)a^4b} + \frac{2}{(3^3)a^3b^2c} \]

Express the denominators as prime factors.

\textbf{LDC of} \( \frac{5}{(3^2)(2)a^4b} \) and \( \frac{2}{(3^3)a^3b^2c} \)

\( (3^2)(2)a^4b \) and \( (3^3)a^3b^2c \)

The LCD is \( (3^3)(2)(a^4)(b^2)(c) \)

Denominators of the rational algebraic expressions

Take the factors of the denominators. When the same factor is present in more than one denominator, take the factor with the highest exponent. The product of these factors is the LCD.

\[ = \frac{5}{(3^3)(2)a^4b} \cdot \frac{3bc}{3bc} + \frac{2}{(3^3)a^3b^2c} \cdot \frac{2a}{2a} \]

\[ = \frac{(5)(3)bc}{(3^3)(2)a^4b^2c} + \frac{(2^2)a}{(3^3)(2)a^4b^2c} \]

\[ = \frac{15bc}{54a^4b^2c} + \frac{4a}{54a^4b^2c} \]

\[ = \frac{15bc + 4a}{54a^4b^2c} \]

Find a number equivalent to 1 that should be multiplied to the rational algebraic expressions so that the denominators are the same with the LCD.
Illustrative example 9: Subtract \( \frac{t + 3}{t^2 - 6t + 9} \) by \( \frac{8t - 24}{t^2 - 9} \).

\[
\frac{t + 3}{t^2 - 6t + 9} - \frac{8t - 24}{t^2 - 9} = \frac{t + 3}{(t - 3)^2} - \frac{8t - 24}{(t - 3)(t + 3)}
\]

The LCD is \((t - 3)^2(t + 3)\)

Express the denominators as prime factors.

\[
\begin{align*}
&= \frac{t + 3}{(t - 3)^2} \cdot \frac{t + 3}{t - 3} - \frac{(8t - 24)}{(t - 3)^2(t + 3)} \cdot \frac{t - 3}{t - 3} \\
&= \frac{(t + 3)(t + 3)}{(t - 3)^2(t + 3)} - \frac{(8t - 24)}{(t - 3)^2(t + 3)} \\
&= \frac{t^2 + 6t + 9}{t^3 - 9t^2 + 27t - 27} - \frac{8t - 48}{t^3 - 9t^2 + 27t - 27} \\
&= \frac{t^2 + 6t + 9 - (8t^2 - 48t + 72)}{t^3 - 9t^2 + 27t - 27} \\
&= \frac{t^2 + 6t + 9 - 8t^2 + 48t - 72}{t^3 - 9t^2 + 27t - 27} \\
&= \frac{-7t^2 + 54t - 63}{t^3 - 9t^2 + 27t - 27}
\end{align*}
\]

Illustrative example 10: Find the sum of \( \frac{2x}{x^2 + 4x + 3} \) by \( \frac{3x - 6}{x^2 + 5x + 6} \).

\[
\frac{2x}{x^2 + 4x + 3} + \frac{3x - 6}{x^2 + 5x + 6} = \frac{2x}{(x + 3)(x + 1)} + \frac{3x - 6}{(x + 3)(x + 2)}
\]

The LCD is \((x + 3)(x + 1)(x + 2)\)

What special products are illustrated in this step?

What property of equality was used in this step?

\[
\begin{align*}
&= \frac{2x}{(x + 3)(x + 1)} \cdot \frac{(x + 2)}{(x + 2)} + \frac{(3x - 6)}{(x + 3)(x + 2)} \cdot \frac{(x + 1)}{(x + 1)} \\
&= \frac{(2x)(x + 2)}{(x + 3)(x + 1)(x + 2)} + \frac{(3x - 6)(x + 1)}{(x + 3)(x + 2)(x + 1)} \\
&= \frac{2x^2 + 4x}{x^3 + 6x^2 + 11x + 6} + \frac{3x^2 - 3x - 6}{x^3 + 6x^2 + 11x + 6}
\end{align*}
\]
Exercises:
Perform the indicated operation. Express your answer in simplest form.

1. \( \frac{3}{x+1} + \frac{4}{x} \)

2. \( \frac{x+8}{x^2-4x+4} + \frac{3x-2}{x^2-4} \)

3. \( \frac{2x}{x^2-9} - \frac{3}{x-3} \)

4. \( \frac{3}{x^2-x-2} - \frac{2}{x^2-5x+6} \)

5. \( \frac{x+2}{x} - \frac{x+2}{2} \)

Activity 12
FLOW CHART

Now that you have learned adding and subtracting rational algebraic expressions. You are now able to fill in the graphic organizer below. Write each step in adding or subtracting rational algebraic expression in each box below.
Rewrite the solution in the first box. Write your solution in the second box. In the third box, write your explanation on how your solution corrects the original one.

<table>
<thead>
<tr>
<th>Original</th>
<th>My Solution</th>
<th>My Explanation</th>
</tr>
</thead>
</table>
| \[
\frac{2}{36 - x^2} - \frac{1}{x^2 - 6x} = \frac{2}{(6 - x)(6 - x)} - \frac{1}{x(x + 6)}
\] = \[
\frac{2}{(x - 6)(x + 6)} - \frac{1}{x(x + 6)}
\] = \[
\frac{2}{x(x - 6)(x + 6)} - \frac{1}{x(x + 6)(x - 6)}
\] = \[
\frac{2x - x + 6}{x(x - 6)(x + 6)}
\] = \[
\frac{x + 6}{x(x - 6)(x + 6)}
\] = \[
\frac{1}{x(x - 6)}
\] = \[
\frac{1}{x^2 - 6x}
\] | | |

<table>
<thead>
<tr>
<th>Original</th>
<th>My Solution</th>
<th>My Explanation</th>
</tr>
</thead>
</table>
| \[
\frac{2}{a - 5} - \frac{3}{a} = \frac{2}{a - 5} - \frac{3a - 5}{a - 5}
\] = \[
\frac{2a}{a - 5(a)} - \frac{3(a - 5)}{a(a - 5)}
\] = \[
\frac{2a}{a - 5(a)} - \frac{3a - 15}{a(a - 5)}
\] = \[
\frac{2a - 3a - 15}{a(a - 5)}
\] = \[
\frac{-a - 15}{a^2 - 5a}
\] | | |
The previous activities deal with the fundamental operations on rational expressions. Let us try these concepts in a different context.

**Activity 14**  
**COMPLEX RATIONAL ALGEBRAIC EXPRESSIONS**

Examine and analyze the following illustrative examples on the next page. Answer the check-up questions.

Rational algebraic expression is said to be in its simplest form when the numerator and denominator are polynomials with no common factors other than 1. If the numerator or denominator, or both numerator and denominator of a rational algebraic expression is also a rational algebraic expression, it is called a complex rational algebraic expression. Simplifying complex rational expression, is transforming it into simple rational expression. You need all the concepts learned previously to simplify complex rational expressions.

<table>
<thead>
<tr>
<th>REVIEW</th>
<th>Perform the operation of the following fractions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\frac{1}{2} + \frac{3}{4} = \frac{1 + 3}{2 + 4}$</td>
<td>4. $\frac{1}{3} \times \frac{5}{9} = \frac{1 \times 5}{3 \times 9}$</td>
</tr>
<tr>
<td>2. $\frac{1}{2} - \frac{3}{4} = \frac{1 \times 4 - 3 \times 2}{2 \times 4}$</td>
<td>5. $\frac{5}{9} + \frac{3}{9} = \frac{5 + 3}{9}$</td>
</tr>
<tr>
<td>3. $\frac{5}{2} - \frac{4}{3} = \frac{5 \times 3 - 4 \times 2}{2 \times 3}$</td>
<td></td>
</tr>
</tbody>
</table>
Illustrative example 11: Simplify \[ \frac{2}{a} \cdot \frac{3}{b} + \frac{5}{b} \cdot \frac{6}{a^2} \].

\[
\frac{2}{a} \cdot \frac{3}{b} + \frac{5}{b} \cdot \frac{6}{a^2} = \frac{(2 \cdot b)}{b \cdot a^2} - \frac{3a}{ab} \\
= \frac{5a^2}{a^2b} + \frac{6b}{a^2b} \\
= \frac{2b - 3a}{ab} \\
= \frac{2b - 3a}{ab} \cdot \frac{a^2b}{5a^2 + 6b} \\
= \frac{2b - 3a}{a^2} \\
= \frac{2ab - 3a^2}{5a^2 + 6b}
\]

Main fraction bar (______) is a line that separates the main numerator and main denominator.

Where did \( \frac{b}{a} \) and \( \frac{a}{b} \) in main numerator and the \( \frac{a^2}{a^2} \) and \( \frac{b}{b} \) in the main denominator come from?

What happens to the main numerator and main denominator?

What principle is used in this step?

Simplify the rational algebraic expression.

What laws of exponents are used in this step?

Illustrative example 12: Simplify \[ \frac{c}{c^2 - 4} - \frac{c}{c - 2} \].

\[
\frac{c}{c^2 - 4} - \frac{c}{c - 2} = \frac{c}{(c - 2)(c + 2)} - \frac{c}{c - 2} \\
= \frac{c}{(c - 2)(c + 2)} - \frac{c}{(c - 2)} \cdot \frac{(c + 2)}{c + 2} \\
= \frac{c + 2}{c + 2} + \frac{1}{c + 2}
\]

Main fraction bar (______) is a line that separates the main numerator and main denominator.
\[
\frac{c}{(c - 2)(c + 2)} - \frac{c(c + 2)}{(c - 2)(c + 2)}
\]
\[
\frac{c + 2}{c + 2} + \frac{1}{(c + 2)}
\]
\[
\frac{c}{(c - 2)(c + 2)} - \frac{c^2 + 2c}{(c - 2)(c + 2)}
\]
\[
\frac{c + 2}{c + 2} + \frac{1}{(c + 2)}
\]
\[
\frac{c - (c^2 + 2c)}{(c - 2)(c + 2)}
\]
\[
\frac{c + 2 + 1}{c + 2}
\]
\[
\frac{-c^2 - 2c + c}{(c - 2)(c + 2)}
\]
\[
\frac{c + 2 + 1}{c + 2}
\]
\[
\frac{-c^2 - c}{(c - 2)(c + 2)}
\]
\[
\frac{c + 2}{c + 2}
\]
\[
\frac{-c^2 - c}{(c - 2)(c + 2)} + \frac{c + 3}{c + 2}
\]
\[
\frac{-c^2 - c}{(c - 2)(c + 2)} \cdot \frac{c + 2}{c + 3}
\]
\[
\frac{(-c^2 - c)(c + 2)}{(c - 2)(c + 2)(c + 3)}
\]
\[
\frac{-c^2 - c}{(c - 2)(c + 3)}
\]
\[
\frac{-c^2 - c}{c^2 + c - 6}
\]

**Exercises**

Simplify the following complex rational expressions.

1. \[
\frac{1}{x} + \frac{1}{y} - \frac{1}{x^2 + \frac{1}{y^2}}
\]
2. \[
\frac{x - y}{x + y} - \frac{y}{x}
\]
3. \[
\frac{1}{b} - \frac{2b}{2b - 2} - \frac{2b}{b - 2} - \frac{3b}{b - 3}
\]
4. \[
\frac{1}{a - 2} - \frac{3}{a - 1}
\]
5. \[
\frac{4 - \frac{4}{y^2}}{2 + \frac{2}{y}}
\]
Activity 15 TREASURE HUNTING

Directions: Find the box that contains the treasure by simplifying rational expressions below. Find the answer of each expression in the hub. Each answer contains direction. The correct direction will lead you to the treasure. Go hunting now.

1. \( \frac{x^2 - \frac{4}{x^2}}{x + \frac{2}{y}} \)
2. \( \frac{x}{2} + \frac{x}{3} - \frac{1}{2} \)
3. \( \frac{3}{x^2 + \frac{3x + 2}{x}} \)

THE HUB

<table>
<thead>
<tr>
<th>( \frac{5x}{3} )</th>
<th>( \frac{x^2 - 2}{x} )</th>
<th>( \frac{1}{x - 1} )</th>
<th>( \frac{x^2 + 2}{x^2 + x - 6} )</th>
<th>( \frac{3}{x^2 + x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 steps to the right</td>
<td>Down 4 steps</td>
<td>3 steps to the left</td>
<td>4 steps to the right</td>
<td>Up 3 steps</td>
</tr>
</tbody>
</table>

Based on the above activity, what are your steps in simplifying complex rational algebraic expressions?
Activity 16  

**VERTICAL CHEVRON LIST**

**Directions:** Make a conceptual map in simplifying complex rational expression using vertical chevron list. Write the procedure or important concepts in every step inside the box. If necessary, add another chevron to complete your conceptual map.

**Web – based Booster:**
Watch the videos in these web sites for more examples

http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_bui1_complexrat.htm

http://www.youtube.com/watch?v=-jli9PP_4HA

http://spot.pcc.edu/~kkling/Mth_95/SectionIII_Rational_Expressions_Equations_and_Functions/Module4/Module4_Complex_Rational_Expressions.pdf

---

Activity 17  

**REACTION GUIDE**

**Directions:** Revisit the second activity. There are sets of rational algebraic expressions in the following table. Check agree if column I is the same as column II and check disagree if the two columns are not the same.

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>Agree</th>
<th>Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{x^2 - xy}{x^2 - y^2} \cdot \frac{x + y}{x^2 - xy} )</td>
<td>( x^{-1} - y^{-1} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{6y - 30}{y^2 + 2y + 1} \div \frac{3y - 15}{y^2 + y} )</td>
<td>( \frac{2y}{y + 1} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{5}{4x^2} + \frac{7}{6x} )</td>
<td>( \frac{15 + 14x}{12x^2} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In this section, the discussion was all about operations on rational algebraic expressions. How much of your initial ideas were discussed? Which ideas are different and need revision? The skills in performing the operations on rational algebraic expressions is one of the key concepts in solving rate-related problems.

What to Understand

Your goal in this section is to relate the operations of rational expressions to real-life problems, especially the rate problems.

Activity 18

WORD PROBLEM

Read the problems below and answer the questions that follow.

1. Two vehicles travelled \((x + 4)\) kilometers. The first vehicle travelled for \((x^2 - 16)\) hours while the second travelled for \(\frac{2}{x - 4}\) hours.
   a. Complete the table below.

<table>
<thead>
<tr>
<th>Vehicles</th>
<th>Distance</th>
<th>Time</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vehicle B</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
b. How did you compute the speed of the two vehicles?
c. Which of the two vehicles travelled faster? How did you find your answer?

2. Jem Boy and Roger were asked to fill the tank with water. Jem Boy can fill the tank in $x$ minutes alone, while Roger is slower by two minutes compared to Jem Boy.
   a. What part of the job can Jem Boy finish in one minute?
   b. What part of the job can Roger finish in one minute?
   c. Jem Boy and Roger can finish filling the tank together within certain number of minutes. How will you represent algebraically, in simplest form, the job done by the two if they worked together?

**Activity 19**

ACCENT PROCESS

List down the concepts and principles in solving problems involving operations of rational algebraic expressions in every step. You can add a box if necessary.

**Activity 20**

PRESENTATION

Present and discuss to the class the process of answering the questions below. Your output will be graded according to reasoning, accuracy, and presentation.

Alex can pour a concrete walkway in $x$ hours alone while Andy can pour the same walkway in two more hours than Alex.
   a. How fast can they pour the walkway if they work together?
   b. If Emman can pour the same walkway in one more hour than Alex, and Roger can pour the same walkway in one hour less than Andy, who must work together to finish the job with the least time?
In this section, the discussion was about application of operations on rational algebraic expressions. It gives you a general picture of relation between the operations of rational algebraic expressions and rate-related problems. What new realizations do you have about the topic? What new connections have you made for yourself? What questions do you still have? Copy the Learned, Affirmed, Challenged cards in your journal notebook and complete each.

**Learned**
What new realizations and learning do you have about the topic?

**Affirmed**
What new connections have you made?
Which of your old ideas have been confirmed/affirmed

**Challenge**
What questions do you still have? Which areas seem difficult for you?
Which do you want to explore
Your goal in this section is to apply your learning in real-life situations. You will be given a practical task which will demonstrate your understanding.

A newly-wed couple plans to construct a house. The couple has already a house plan made by their engineer friend. The plan of the house is illustrated below:

As a foreman of the project, you are tasked to prepare a manpower plan to be presented to the couple. The plan includes the number of workers needed to complete the project, their daily wage, the duration of the project, and the budget. The man power plan will be evaluated based on reasoning, accuracy, presentation, practicality and efficiency.
## Rubrics for your output

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>Outstanding 4</th>
<th>Satisfactory 3</th>
<th>Developing 2</th>
<th>Beginning 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>All computations are correct and shown in detail.</td>
<td>All computations are corrects.</td>
<td>Most of the computations are correct.</td>
<td>Some of the computations are correct.</td>
</tr>
<tr>
<td>Presentation</td>
<td>The presentation is delivered in a very convincing manner. Appropriate and creative visual materials were used.</td>
<td>The presentation is delivered in a clear manner. Appropriate visual materials were used.</td>
<td>The presentation is delivered in a disorganized manner. Some visual materials.</td>
<td>The presentation is delivered in a clear manner. It does not use any visual materials.</td>
</tr>
<tr>
<td>Practicality</td>
<td>The proposed plan will be completed at the least time.</td>
<td>The proposed plan will be completed in lesser time.</td>
<td>The proposed project will be completed with greater number of days.</td>
<td>The proposed plan will be completed with the most number of days.</td>
</tr>
<tr>
<td>Efficiency</td>
<td>The cost of the plan is minimal.</td>
<td>The cost of the plan is reasonable.</td>
<td>The cost of the plan is expensive.</td>
<td>The cost of the plan is very expensive.</td>
</tr>
</tbody>
</table>
SUMMARY

Now that you have completed this module, let us summarize what you have learned:

1. Rate – related problems can be modelled using rational algebraic expressions.
2. Rational algebraic expression is a ratio of two polynomials where the denominator is not equal to one.
3. Any expression raised to zero is always equal to one.
4. When an expression is raised by a negative integer, it is the multiplicative inverse of the expression.
5. Rational algebraic expression is in its simplest form if there is no common factor between numerator and denominator except 1.
6. To multiply rational algebraic expression, multiply the numerator and denominator then simplify.
7. To divide rational algebraic expression, multiply the dividend by the reciprocal of the divisor then multiply.
8. To add/subtract similar rational algebraic expressions, add/subtract the numerators and copy the common denominator.
9. To add/subtract dissimilar rational algebraic expressions, express each expression into similar one then add/subtract the numerators and copy the common denominator.
10. Complex rational algebraic expression is an expression where the numerator or denominator, or both numerator and denominator are rational algebraic expressions.

GLOSSARY USED IN THIS LESSON

Complex rational algebraic expression – an expression where the numerator or denominators or both numerator and denominator are rational algebraic expressions.

LCD – also known as Least Common Denominator is the least common multiple of the denominators.

Manpower plan – a plan where the number of workers needed to complete the project, wages of each worker in a day, how many days can workers finish the job and how much can be spend on the workers for the entire project.

Rate – related problems – Problems involving rates (e.g., speed, percentage, ratio, work)

Rational algebraic expression – ratio of two polynomials where the denominator is not equal to one.
REFERENCES AND WEBSITE LINKS USED IN THIS MODULE:

Learning Package no. 8, 9, 10, 11, 12, 13. Mathematics Teacher’s Guide, Funds for assistance to private education, 2007
Orines, F., Diaz, Z., Mojica, M., Next century mathematics intermediate algebra, Phoenix Publishing House, Quezon Ave., Quezon City 2007
Oronce, O., Mendoza, M., e – math intermediate algebra, Rex Book Store, Manila, Philippines, 2010
http://www.youtube.com/watch?v=-jli9PP_4HA
http://spot.pcc.edu/~kkling/Mth_95/SectionIII_Rational_Expressions_Equations_and_Functions/Module4/Module4_Complex_Rational_Expressions.pdf
Image credits
http://www.portlandground.com/archives/2004/05/volunteers_buil_1.php