I. INTRODUCTION AND FOCUS QUESTIONS

REASONING CONCLUSION

Do you think it is possible to make a valid conclusion without even going through the process of investigation? What would you do if you were asked to make a decision that will affect many people? Many aspects in our life involve decisions and proof.

Geometry deals with reasoning and how to prove something. It gives us the logical way on how to prove a certain thing. In this module you will find the answer to the questions: “How to make a valid conclusion?” and “When to use inductive and deductive proofs?”

II. LESSONS AND COVERAGE

In this module, you will examine this question when you go through the following lessons:

- **Lesson 1** – If-then Statements
- **Lesson 2** – Inductive and Deductive Reasoning
- **Lesson 3** – Writing Proof

In these lessons, you will learn to:

| Lesson 1 | • Identify the hypothesis and conclusions of if-then and other types of statements.  
|          | • Formulate the inverse, converse, and contrapositive of an implication. |
| Lesson 2 | • Distinguish between inductive and deductive reasoning. |
**Lesson 3**

- Provide formal arguments that explain results of a phenomenon or a situation.
- Use syllogism in writing formal arguments as a series of statements that make up a proof.
- Explain the need and importance for defined terms previously learned.
- Differentiate between postulate and theorem and give the importance of each.

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**Module Map**

Here is a simple map of the lessons that will be covered in this module:

- If - then Statements
- Inductive and Deductive Reasoning
- Writing Proof

**EXPECTED SKILLS:**

To do well in this module, you need to remember and do the following:

1. Define terms that are unfamiliar to you.
2. Explore websites that would be of great help for your better understanding of the lessons.
3. Take down notes of the important concepts in your journal.
4. Perform and complete the exercises provided.
5. Collaborate with the teacher and peers.
III. PRE-ASSESSMENT

Choose the letter of the correct answer

1. Which of the following best describes deductive reasoning?
   a. using logic to draw conclusions based on accepted statements
   b. accepting the meaning of a term without definition
   c. defining mathematical terms in relation with physical objects
   d. inferring a general truth by examining a number of specific examples

2. Theorem: A triangle has at most one obtuse angle. Francisco is proving the theorem above by contradiction. He began by assuming that in $\triangle ABC$, $\angle A$ and $\angle B$ are both obtuse. Which theorem will Francisco use to reach a contradiction?
   a. If two angles of a triangle are congruent, the sides opposite the angles are congruent.
   b. If two supplementary angles are congruent, each angle measures 90°.
   c. The largest angle in a triangle is opposite the longest side.
   d. The sum of the measures of the angles of a triangle is 180°.

3. If $m\angle R + m\angle M = 90°$ then
   a. $\angle R \cong \angle M$.
   b. $\angle R$ and $\angle M$ are right angles.
   c. $\angle R$ and $\angle M$ are complementary.
   d. $\angle R$ and $\angle M$ are supplementary.

4. The converse of the statement: "if you are in love then you are inspired", is,
   a. If you are not in love, then you are not inspired.
   b. If you are inspired, then you are in love.
   c. If you are not inspired, then you are not in love.
   d. If you are in love, you are not inspired.

5. The if-then form of the statement: "Parallel lines never intersect", is:
   a. If two lines intersect, then they are parallel.
   b. If two lines are parallel, then they never intersect.
   c. If two lines are not parallel then they intersect.
   d. If two lines intersect, then they are not parallel.
6. What is the inverse of the statement, "If the number is divisible by 2 and 3, then it is divisible by 6".
   a. If the number is divisible by 6, then it is divisible by 2 and 3.
   b. If the number is not divisible by 2 and 3, then it is not divisible by 6.
   c. If the number is not divisible by 6, then it is not divisible by 2 and 3.
   d. If the number is divisible by 2 and 3, then it is not divisible by 6.

7. What property is illustrated in: If $\angle A \cong \angle B, \angle B \cong \angle C$ then $\angle A \cong \angle C$.
   a. Reflexive Property
   b. Symmetric Property
   c. Transitive Property
   d. Addition Property

8. Using the distributive property,
   $4(a + b) = \underline{\hspace{2cm}}$.
   a. $4a + b$
   b. $B + 4a$
   c. $4a + 4b$
   d. $4 + a + b$

9. Supply a valid conclusion for the given hypothesis: if $\overrightarrow{OM}$ bisects $\angle LON$ then
   a. $\angle LOM \cong \angle NOM$
   b. $\angle LOM \cong \angle LON$
   c. $\angle MON \cong \angle NOL$
   d. $m\angle LON = m\angle LOM + m\angle MON$

10. The method of proof by contradiction is:
    a. direct proof
    b. formal proof
    c. indirect proof
    d. two column proof

11. If garbage are disposed properly then dengue diseases will be prevented. What is the underlined portion called in the conditional statement?
    a. the conclusion
    b. the hypothesis
    c. the argument
    d. the converse
12. How many dots are there in the three figure?

Figure 1

Figure 2

Figure 3

13. Which of the following statements is true?
   a. If $\angle 1$ has a measure of $90^\circ$, then $\angle 1$ is obtuse.
   b. If $\angle 1$ has a measure of $140^\circ$, then $\angle 1$ is acute.
   c. If $\angle 1$ has a measure of $35^\circ$, then $\angle 1$ is acute.
   d. If $\angle 1$ has a measure of $180^\circ$, then $\angle 1$ is right.

14. Which of the following statements is false?
   a. Any four non-collinear points lie in a distinct plane.
   b. A plane contains at least 3 non-collinear points.
   c. Any two lines intersect at a point.
   d. Through two given points we can draw three lines.

15. Rewrite the statement in if-then form.
   a. A figure has four sides if and only if it is a quadrilateral.
   b. If a figure is a quadrilateral, then it has four sides.
   c. If a figure has four sides, then it is a quadrilateral.
   d. A figure is a quadrilateral if and only if it has four sides.

16. Name the property which justifies the following conclusion.

Given: $JB = 28$
Conclusion: $JB + 4 = 32$

   a. Addition property of equality
   b. Multiplication property of equality
   c. Substitution property of equality
   d. Transitive property of equality
For 17-20 Give the answer.

17. $\angle 1$ and $\angle 2$ are complementary angles. $\angle 1$ and $\angle 3$ are vertical angles. If $m\angle 3 = 49^\circ$, find $m\angle 2$.

18. What is the missing reason in the following proof?

$m\angle 1 = m\angle 3, \ m\angle 2 = m\angle 3$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1. m\angle 1 = m\angle 3, m\angle 2 = m\angle 3$</td>
<td>1. given</td>
</tr>
<tr>
<td>$2. m\angle 2 = m\angle 3$</td>
<td>2. <em><strong>?</strong></em>_</td>
</tr>
</tbody>
</table>

19. Supply the missing statement in the following proof.

Given: $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$
Prove: $m\angle 1 = m\angle 3$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1. m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$</td>
<td>1. given</td>
</tr>
<tr>
<td>$2. $________<strong>?</strong>__________$</td>
<td>2. Reflexive Property</td>
</tr>
<tr>
<td>$3. m\angle 1 = m\angle 3$</td>
<td>3. Subtraction property</td>
</tr>
</tbody>
</table>

20. What conclusion can be logically deduced based on the following statements?
If you are catholic, you are against the RH Bill
Mrs. Romano is a Catholic.
Let's begin this lesson by accomplishing the activity sheet below called INBOX – OUTBOX sheet.

**Activity 1**

**INBOX – OUTBOX SHEET**

**Description:** This activity is intended to elicit your prior knowledge regarding the lesson.

**Direction:** Answer the question below and write your answer in the space provided IN THE BOX.

How do you make valid conclusions if faced with problems in life such as having failing grades, meeting deadlines and even in love-life troubles?
**Description:** There are things in life which involve decision-making. Find out how valid decision making affects our life. The given article below deals with the effect of having or giving misguided conclusion.

**Direction:** Read the excerpts on the article from Bombo Radio Philippines entitled, "Judge sinibak ng SC due to wrong decisions" then answer the follow – up questions below.

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**Judge sinibak ng SC due to wrong decisions**

Revoked of license by the Supreme Court (SC) is a judge in Cotabato City because of the issuance of a decision in the case of annulment of marriage without conducting a hearing.

In the per curiam decision of the Supreme Court en banc discharged on May 15, 2012, proven guilty of gross misconduct and dishonesty is Judge Cader Indar Al Haj, presiding judge of the Regional Trial Court (RTC), Branch 14, Cotabato City and also serve as acting presiding judge of RTC, Branch 15, Shariff Aguak, Maguindanao.

Along with its disbarment Indar was removed from the service and also the benefits that he should get from his retirement except leave credits.

Also stated in the decision was Indar can’t work in any office of the government owned including any government owned and controlled corporations.

According to the court, Indar violated the Canons 1 and 7, as well as Rule 1.01 of the Code of Professional Responsibility, thus, also removing his name on the roll of attorneys.

The penalty of Indar stemmed from a report sent by the LCR of Manila and Quezon City to the Office of Court Administration (OCA) in relation to the decisions resolutions and orders in the marriage annulment issued by the same judge.


This site provided the article above entitled "Judge sinibak ng SC due to wrong decisions"
QUESTIONS

1. Comment on the article.
2. Site a situation where important decision making is needed.
3. Suggest a procedure on how to make a wise decision.

You gave your initial ideas on how to make sound judgment and how useful it was. Let’s now find out the other answers by doing the next part. What you have learned in the next sections will also enable you to do the final project which involves investigating mathematical concepts.

What to Process

Your goal in this section is to learn and understand key concepts of reasoning and proving. You will be dealing with If-Then statement, Deductive and Inductive reasoning, and writing proofs.

Activity 3 JUDGE US!

Description: A lot in the statements that we encounter are logically constructed or but NOT valid or acceptable. This activity deals with determining which statement is valid or not.

Direction: From the given statements tell whether the statement is valid or not.

1. Students who are good in mathematics are smart.
   Enchong is smart, then he is good in mathematics.
2. Young actresses are health conscious.
   Kim is a young actress then she is health conscious.
3. If it rains then the sports fest will be cancelled.
   It rains therefore the sports fest is cancelled.
4. If the lines are parallel they do not intersect
   Line $x$ and line $y$ do not intersect; therefore, they are parallel.
5. If two angles are right angles, then they are congruent.
   $\angle A$ and $\angle B$ are congruent, then they are right angles.

For items 6 to 10 complete the statement and justify your answer

6. Miss Earth candidates are environmentalists.
   Miss Jaybee is a candidate to the Miss Earth search, therefore
7. If you are at SM you got it all. Marie is at SM then
8. If you bank with BDO they find ways. Vincent has deposit at BDO then
9. If you drink coke you find happiness. Jay is drinking coke then
10. Globe connects people. Dedeth is using Globe simcard then

**Questions**

a. What have you noticed about the statements given above?
b. Take one of the statements and tell something about it.
c. What is common to all of the statements?

Write your answers in your journal and have a small discussion with your group.

**You have just encountered conditional statements or the If-then statements.**

An *if-then* statement is composed of two clauses: the *if-* clause and the *then-* clause. We can denote a letter for each clause, $p$ for the *if* clause and $q$ for the *then* clause. The statement is in the form “If $p$ then $q$. Conditional statements are formed by joining two statements $p$ and $q$ using the words *if* and *then*. The $p$ statement is called the *hypothesis* and the $q$ statement is the *conclusion*.

A simple flow of reasoning from if-clause to the then-clause is called simple *implication*.

There are some conditional statements not written in this form but you can rewrite them using the if-then form. How will you identify the hypothesis and the conclusion? You try this:

1. Cigarette smoking is dangerous to your health.
   *If-then form______________________________*
   *Hypothesis______________________________*
   *Conclusion______________________________*

2. It is more fun in the Philippines.
   *If-then form______________________________*
   *Hypothesis______________________________*
   *Conclusion______________________________*

3. A segment has exactly one midpoint.
   *If-then form______________________________*
   *Hypothesis______________________________*
   *Conclusion______________________________*
4. Angles in a linear pair are supplementary.
   If-then form ______________________________________
   Hypothesis ________________________________________
   Conclusion ________________________________________

5. Vertical angles are congruent.
   If-then form ______________________________________
   Hypothesis ________________________________________
   Conclusion ________________________________________

How do you distinguish the hypothesis from the conclusion when the statement is not in the if-then form? See the examples below.

1. National Disaster Risk Reduction Council volunteers are busy during calamities.
2. An eighteen year old Filipino can cast his/her vote during election.
3. All right angles are congruent.
4. Three non-collinear points determine a plane.
5. Perpendicular lines are intersecting lines.

Discuss with a partner the underlined part of the sentence and the one in bold letters. What part of the sentence are the underlined words? What part of the sentence are in bold letters? Which is the hypothesis and which is the conclusion? Rewrite the statements to if-then form.

Now that you know what conditional statements are, and you can identify the hypothesis and the conclusion, have more practice in answering the exercises below.

Exercise 1

Convert each statement in if-then form, then identify the hypothesis and the conclusion.

1. Opposite sides of a rectangle are parallel.
2. Filipinos are God-fearing people.
3. The sum of the measures of complementary angles is 90°.
4. Good citizens obey rules and regulations.
5. A triangle is a polygon of three sides.
6. A quadrilateral has four sides.
7. Two points determine a line.
8. The intersection of two lines is a point.
9. Two intersecting lines line in one plane.
10. The sum of the angles forming a linear pair is 180°

Now that you are well-versed in converting conditional statement to if-then form, you can easily identify the hypothesis and the conclusion. When do you say that the implication is true or false?
The implication \( p \to q \) is always true except in the case that \( p \) is true and \( q \) is false. See the truth table for implication below.

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \to q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
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<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

*This time let us make another statement from the given one. Let us do the activity.*

**Activity 4 JUMBLED WORDS**

**Direction:** Make a sentence from the jumbled words.

**POLYGON TRIANGLE A IS A**

1. A TRIANGLE IS A POLYGON.
2. A POLYGON IS A TRIANGLE.

Let’s take the #1 sentence as our first statement. First we can convert it to if-then form, then we can form its converse, inverse, and contrapositive.

Study the table below.

<table>
<thead>
<tr>
<th>Statement</th>
<th>If-then form</th>
<th>Converse</th>
<th>Inverse</th>
<th>Contrapositive</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A triangle is a polygon.</td>
<td>If an object is a triangle, then it is a polygon</td>
<td>If an object is a polygon, then it is a triangle.</td>
<td>If an object is not a triangle, then it is not a polygon.</td>
<td>If an object is not a polygon, then it is not a triangle.</td>
</tr>
</tbody>
</table>

Discuss with your group how the converse is written? Inverse? and contrapositive of a given statement.

If \( p \) is ; If the object is a triangle
\( q \) is : then it is a polygon

What happen to \( p \) and \( q \) in the converse?
Compare the inverse and the original statement. What did you do with \( p \)? what did you do with \( q \)?
Observe the changes in the contrapositive.
Summarize your observation in terms of $p$ and $q$.

Let’s take another statement: **An even number is divisible by two**.

If-then form

Converse

Inverse

Contrapositive

We can summarize how to convert the statement in terms of $p$ and $q$. See the table below.

<table>
<thead>
<tr>
<th>Statement</th>
<th>If $p$, then $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converse</td>
<td>If $q$, then $p$</td>
</tr>
<tr>
<td>Inverse</td>
<td>If not $p$, then not $q$</td>
</tr>
<tr>
<td>Contrapositive</td>
<td>If not $q$, then not $p$</td>
</tr>
</tbody>
</table>

Exercise 2

A. Fill up the table below.

<table>
<thead>
<tr>
<th>Statement</th>
<th>If two angles are congruent, then they have the same measure.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converse</td>
<td></td>
</tr>
<tr>
<td>Inverse</td>
<td></td>
</tr>
<tr>
<td>Contrapositive</td>
<td></td>
</tr>
</tbody>
</table>

B. State the converse of the following statements:

1. Three non-collinear points determine a plane.
2. A rectangle has four right angles.
3. Perpendicular lines intersect.

Go back to Activity 4.

If $p$, then $q$:

If an object is a triangle then it is a polygon.

Converse

If $q$, then $p$:

If an object is a polygon then it is a triangle.

Analyze the converse. Is it true? If not, give a counter example.

The converse is false because square is a a polygon. It is not a triangle.
If \( p \), then \( q \): If a number is even then it is divisible by two.
If \( q \), then \( p \): If a number is divisible by two then it is even.

The converse is true.

Try to analyze the converse of the statements in B.
So what can you conclude about the converse of a statement?
Is the converse of a given statement always true?

### Activity 5 “PICTURE ME”

Observe the set of pictures. Translate the pictures into conditional statements. State the converse, inverse and contrapositive of the conditional statements.
Classify each us true or false and justify.
Go to other group, share each other’s answers and come up with a common conclusion.

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Now that you can identify the hypothesis and the conclusion in the if-then statement, and form its converse, you are now ready to study the kinds of reasoning in the next section.
Lesson 2

Inductive and Deductive Reasoning

Activity 6  WHY OH WHY?

Each group will be given this activity sheet to accomplish.

1. Look carefully at the figures, what is next?

   ![Figures]

2. Study the pattern. Draw the next figure.

   ![Pattern]

3. My math teacher is strict.
   My previous math teacher was strict.
   What can you say about all math teachers?

4. 
   
   1 × 10 = 10
   2 × 10 = 20
   3 × 10 = 30
   5 × 10 = 50
   24 × 10 = 240
   2345 × 10 = ______.

5. Every time Jackie visits her doctor she receives excellent services. With this she believes that._______________________

Discuss the following with your group
• How did you arrive at your answer?
• Did you agree at once on your answer?
• Were there arguments among the members?
• What you have shown is inductive reasoning. Give 3 examples of inductive reasoning.
• Based on the activity, define inductive reasoning?

Inductive reasoning takes specific examples to make a general rule.
Complete the table below.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Filipinos are hospitable. Bonifacio is a Filipino.</td>
<td></td>
</tr>
<tr>
<td>2. If points are collinear, then they lie on the same plane</td>
<td></td>
</tr>
<tr>
<td>Points R, M, and N are collinear.</td>
<td></td>
</tr>
<tr>
<td>3. A quadrilateral is a polygon of four sides</td>
<td></td>
</tr>
<tr>
<td>4. Smoking can cause cancer. Tomas is smoking</td>
<td></td>
</tr>
<tr>
<td>5. An angle is acute if its measure is between 0° and 90°. Angle B is acute.</td>
<td></td>
</tr>
</tbody>
</table>

You have just encountered Deductive reasoning. Can you give the difference between inductive and deductive reasoning?

Deductive reasoning is reasoning which begins using basic and general statements to prove more complicated statements.

Inductive reasoning is judging by experience while deductive reasoning is judging by logical progression.

**Exercise 3**

Draw conclusion from each given situation and identify the kind of reasoning used.

1. 5, 10, 15, 20. The next number is ____.
2. Coplanar points are points on the same plane. X, Y, Z are coplanar.
   Therefore ____________________
3. Regular polygon is equilateral. BELEN is a regular pentagon.
   Therefore ____________________
4. A child’s teacher in pre school was a female. In his grades 1 and 2 his teachers were both a female. The child may say ____________
5. Filipinos are peace-loving people. Julia is a Filipino. Therefore ________

The main focus in the study of geometry is to learn how to think logically. Do you still remember the If-then statement? Which one is the hypothesis? the conclusion?
The parts of a deductive reasoning are:
• Hypothesis – the statement which is accepted or known at the beginning
• Conclusion – the statement drawn from the hypothesis.

Activity 8  LET'S CONCLUDE

A. Supply the conclusion for the given hypothesis
1. If \( \angle 1 \equiv \angle 2 \), then __________
2. If \( AB = CE \), then __________
3. If \( \angle B \) and \( \angle E \) are complementary, then __________
4. \( m\angle 3 + m\angle 5 = 180 \), then __________
5. If \( \angle A \) and \( \angle X \) form a linear pair, then __________

B. Supply a valid conclusion for the given hypothesis on the first blank and the corresponding reason on the second blank
6. If \( \angle B \) is a right angle,
   Then _______________ ______________
7. If \( m\angle 3 + m\angle 4 = 180 \)
   Then _______________ ______________
8. If \( PM \) bisects \( \angle APO \)
   Then _______________ ______________
9. If \( BP \perp BC \)
   Then _______________ ______________
10. \( \triangle BOS \) is isosceles.
    Then _______________ ______________

From the hypothesis we derive another statement that is the conclusion Where did you base your conclusion? Have you recalled your undefined terms, definitions and postulates? They will play a very important role in our next section.
Lesson 3

Proving Theorems

In proving theorems, the properties of equality and congruence are the bases for reasoning.

Properties of Equality

Addition Property of Equality (APE)
For all real numbers \(a, b, c\) and \(d\), if \(a = b\) and \(c = d\), then \(a + c = b + d\)

Subtraction Property of Equality (SPE)
If \(a = b\) and \(c = d\), then \(a - c = b - d\).

Multiplication Property of Equality (MPE)
If \(a = b\), then \(ac = bc\)

Division Property of Equality (DPE)
If \(a = b\) then \(ac = bc\)

Substitution Property of Equality
If \(m\angle A = 60\), \(m\angle B = 60\) then \(m\angle A = m\angle B\)

Distributive Property
\(a(b + c) = ab + ac\)

Properties of Congruence

Reflexive Property
\(\overline{AB} \cong \overline{AB}\)

Symmetric Property
If \(\angle A \cong \angle B\) then \(\angle B \cong \angle A\)
Transitive Property

If $\angle A \cong \angle B$ and $\angle B \cong \angle C$ then $\angle A \cong \angle C$

Aside from the properties of equality and congruence, you should be equipped with the knowledge about undefined terms, definitions, and postulates in geometry. These are necessary to successfully support the statement of a proof.

Exercise 4
Justify each statement by giving the Property of Equality or Property of Congruence used.

If $TX = BK$, then $BK = TX$
$8(m + n) = 8m + 8n$
If $CT = 12$ and $PR + CT = 20$, then $PR + 12 = 20$.
$m\angle HIT = m\angle HIT$
If $\angle S \cong \angle P$, $\angle B \cong \angle S$, then $\angle P \cong \angle B$

One of the tools used in proving is reasoning, specifically deductive reasoning. Deductive reasoning is a type of logical reasoning that uses accepted facts to reason in a step-by-step manner until we arrive at the desired statement.

A proof is a logical argument in which each statement you make is supported/justified by given information, definitions, axioms, postulates, theorems, and previously proven statements.

Remember:

- **Postulate** is a statement that is accepted without proof.
- **Theorem** is a statement accepted after it is proved deductively.

In proving theorems you have to follow these steps:

- Read and understand the theorem
- Label the hypothesis as given and the conclusion as Prove
- Draw the figure and label the parts correctly.
- Write the proof which consists of the statements and reasons.
Proofs can be written in different ways

1. **Paragraph Form/ Informal Proof:**

   The paragraph or informal proof is the type of proof where you write a paragraph to explain why a conjecture for a given situation is true.

   **Given:** \( \angle LOE \) and \( \angle EOV \) are complementary

   **Prove:** \( \overline{LO} \perp \overline{OV} \)

   **Proof:** Since \( \angle LOE \) and \( \angle EOV \) are complementary, then \( m\angle LOE + m\angle EOV = 90^\circ \) by definition of complementary angles. Thus, \( m\angle LOE + m\angle EOV = m\angle LOV \) by angle addition postulate and \( m\angle LOV = 90^\circ \) by transitive property of equality. So, \( \angle LOV \) is a right angle by definition of right angles; therefore, \( \overline{LO} \perp \overline{OV} \) by definition of perpendicularity.

2. **Two-Column Form/ Formal Proof:**

   **Given:** \( m\angle SEP = m\angle TER \)

   **Prove :** \( m\angle 1 = m\angle 3 \)

   **Write the missing reasons**

   **Proof:**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( m\angle SEP = m\angle TER )</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( m\angle SEP = m\angle 1 + m\angle 2 )</td>
<td>2. Angle Addition Postulate</td>
</tr>
<tr>
<td>3. ( m\angle TER = m\angle 2 + m\angle 3 )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3 )</td>
<td>4. Substitution Property</td>
</tr>
<tr>
<td>5. ( m\angle 2 = m\angle 2 )</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( m\angle 1 = m\angle 3 )</td>
<td>6. Subtraction Property</td>
</tr>
</tbody>
</table>

   Study carefully the parts, especially the proof. How do we derive the statements and the reasons.

   Try the flow chart form using the same example.
3. **Flowchart Form:**

A **flowchart-proof** organizes a series of statements in a logical order, starting with the given statements. Each statement together with its justification is written in a box. Arrows are used to show how each statement leads to another. It can make ones logic visible and help others follow the reasoning.

**Example 1**

**Flow Chart Proof**

Given: \( RA \cong RE \)

\( CE \cong CA \)

Prove: \( \angle E \cong \angle A \)

1. \( RA \cong RE \)

2. \( CE \cong CA \)

3. \( RC \cong RC \)

4. \( \triangle RAC \cong \triangle REC \)

5. \( \angle E \cong \angle A \)

**Example 2**

**CPCTC**

\( m\angle SEP = m\angle TER \)

Given: Angle Addition Postulate

\( m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3 \)

Substitution

\( m\angle 1 = m\angle 3 \)

Reflexive Property

\( m\angle TER = m\angle 2 + m\angle 3 \)

Angle Addition Postulate

\( m\angle SEP = m\angle 1 + m\angle 2 \)

You might want to watch a video lesson on this kind of proof, you may visit the following link:

http://www.youtube.com/watch?feature=player_embedded&v=3Ti7-Ojr7Cg
4. **Indirect Proof**

An indirect proof usually is paragraph form, the opposite of the statement to be proven is assumed true until the assumption leads to contradiction.

*Example:*

Given: \( \triangle BEL \) is isosceles triangle with vertex \( \angle B \)

Prove: \( \angle B \cong \angle L \)

Proof: Assume that \( \angle B \cong \angle L \)

Given that \( \triangle BEL \) is isosceles therefore

\( \overline{BE} \cong \overline{BL} \) by the definition of isosceles triangle

\( \angle B \cong \angle L \) because if two sides of a triangle are congruent then the angles opposite these sides are congruent; thus, the assumption is false and therefore \( \angle B \cong \angle L \).

**Exercise 5**

*Work in group*

Prove that if two parallel lines are cut by a transversal, then the alternate interior angles are congruent. Discuss and show the proof.

1. Given:________________

Prove: ____________

Figure:

Proof:

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Show the proof of the following.

2. Given: \( \angle 1 \cong \angle 2 \)

\( \angle 3 \cong \angle 4 \)

Prove: \( \triangle COD \cong \triangle BOD \)

3. Given: \( \overline{OP} \perp \overline{PE}; \overline{EB} \perp \overline{PE} \)

*\( T \) is the midpoint of \( \overline{PE} \)*

Prove: \( \overline{OP} \cong \overline{BE} \)
In this section, the discussion was about proofs.

Go back to the previous section and compare your initial ideas with the ideas discussed. How much of your initial ideas are found in the discussion? Which ideas are different and need revision?

Now that you know the important ideas about this topic, let’s go deeper by moving on to the next section.

**What to Understand**

What will you do before jumping into a conclusion?

*This is a picture analysis activity, where you can form conditional statements out of the picture. Proceed by forming another statement drawn from the original one.*

**Activity 9 “CASE SOLVED”**

From the sets of geometric representations on real-life situations problems, write your reasons inductively or deductively on the "Reason Out Activity Sheet".

Be ready to present your arguments to persuade your classmates.

Your arguments will be rated in terms of coherence, mathematical thinking and conclusions made.
The city’s newspaper will release its 2nd volume this year. It contains a new column that will explain phenomena that deals with the concepts of mathematics. As a contributor writer, you were tasked to write an article that will explain the “phenomenon of perspective” it shows that when two parallel lines are seen from a far, the lines intersect. Your article will be evaluated by the head writer and editor-in-chief based on its coherence, mathematical thinking, and conclusions made.

In this section, the discussion was about proofs.

What new realizations do you have about the topic? What new connections have you made for yourself?

Now that you have a deeper understanding of the topic, you are ready to do the tasks in the next section.

In this section, you will be applying your understanding of inductive and deductive reasoning through the following activity that reflect meaningful and relevant situations.

Math Magazine will release its November issue themed “MATH INVESTIGATES”. As one of the investigators you were tasked to make a mathematical investigation that will enlighten the readers by providing valid conclusions. The written output of your investigation will be presented to the Head writer, writers, and Editor-In-Chief and shall be evaluated based on to its coherence, clarity, judgment, and mathematical reasoning.

In this section, your task was on math investigations.

How did you find the performance task? How did the task help you see the application use of the topic?
You have just completed this lesson. Before you go to the next lesson, you have to answer the following post-assessment.

**Rubric on Mathematical Investigation**

<table>
<thead>
<tr>
<th>CRITERIA</th>
<th>Outstanding 4</th>
<th>Satisfactory 3</th>
<th>Developing 2</th>
<th>Beginning 1</th>
<th>RATING</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematical Reasoning</strong></td>
<td>The explanation is in-depth and comprehensive. The use of appropriate and sufficient statements is extremely evident in the process of proving.</td>
<td>Explanation is comprehensive. The use of appropriate and sufficient statements is evident in the process of proving.</td>
<td>Explanation is vague but the use of appropriate and sufficient statements evident in the process of proving.</td>
<td>Explanation is vague and use of appropriate and sufficient statements is NOT evident in the process of proving.</td>
<td></td>
</tr>
<tr>
<td><strong>Coherence</strong></td>
<td>Acceptable statements are logical and there are no errors in the process of proving. The procedures were strictly utilized.</td>
<td>Acceptable statements are logical and there are no errors in the process of proving.</td>
<td>Acceptable statements are logical but there are minor errors in the process of proving.</td>
<td>Acceptable statements are NOT logical and there are major errors in the process of proving.</td>
<td></td>
</tr>
<tr>
<td><strong>Clarity</strong></td>
<td>The proving process and acceptable statements are clearly written which facilitates a straightforward understanding.</td>
<td>The proving process is clearly written which facilitates a straightforward understanding.</td>
<td>Some parts in the proving process is somehow unclearly written which do not facilitate a straightforward understanding.</td>
<td>All parts of the proving process is vaguely written which do not facilitate a straightforward understanding.</td>
<td></td>
</tr>
<tr>
<td>Conclusion</td>
<td>A valid and acceptable conclusion was derived from the result of the proving thus, was able to give enlightenment to others and create an innovative statement/formula for the future use in Mathematics.</td>
<td>A valid and acceptable conclusion was derived from the result of the proving thus, was able to give enlightenment to others.</td>
<td>A valid and acceptable conclusion was derived from the result of the proving but was moderately able to give enlightenment to others.</td>
<td>A invalid and unacceptable conclusion was derived from the result of the proving thus, unable to give enlightenment to others.</td>
<td>OVERALL RATING</td>
</tr>
</tbody>
</table>